On Spatial Reasoning with Description Logics — Position Paper

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Abstract

We discuss a family of DLs called $\mathcal{ALCI}_{\mathcal{RCC}}$ which are suitable for qualitative spatial reasoning on various levels of granularity. In contrast to our previous work where we investigated concept satisfiability in the basic description logic $\mathcal{ALC}$ in combination with composition–based role inclusion axioms of the form $S \circ T \sqsubseteq R_1 \sqcup \cdots \sqcup R_n$ and role disjointness, we are now only considering the role axioms that are derived from the so-called RCC composition tables. In order to correctly capture the semantics of these relationships, inverse and disjoint roles are needed. We discuss what we have found out so far. We make some remarks on finite model reasoning, which is especially useful in database applications; e.g. the deductive qualitative Geographic Information System (GIS) scenario we have in mind.

1 Introduction and Motivation

At DL 2001, we presented an overview of various $\mathcal{ALC}$-extensions with composition based role inclusion axioms of the form $S \circ T \sqsubseteq R_1 \sqcup \cdots \sqcup R_n$, enforcing $S^{\mathcal{I}} \circ T^{\mathcal{I}} \subseteq R_1^{\mathcal{I}} \cup \cdots \cup R_n^{\mathcal{I}}$ on the models $\mathcal{I}$ (see [4]). A set of these role axioms was called a role box, and the resulting logic was called $\mathcal{ALC}_{\mathcal{RA}^{\ominus}}$. In previous work we have shown that concept satisfiability in $\mathcal{ALC}_{\mathcal{RA}^{\ominus}}$ (and even in smaller sublanguages) becomes undecidable if concept satisfiability w.r.t. arbitrary role boxes is considered. However, certain classes of so-called "admissible" role boxes satisfying additional conditions were shown to be decidable (e.g. the logic we called $\mathcal{ALC}_{\mathcal{RASG}}$). One of the original motivations for extending $\mathcal{ALC}$ in that way was to augment a description logic like $\mathcal{ALC}$ with some kind of qualitative spatial reasoning capabilities (see also [2]). Since role disjointness is an important requirement if one considers roles as *spatially exclusive base relationships,* we also investigated the logic $\mathcal{ALC}_{\mathcal{RA}}$, enforcing role disjointness on all roles ($R, S \in \mathcal{N_R}$, $R \neq S$: $R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$). $\mathcal{ALC}_{\mathcal{RA}}$ turned out to be undecidable as well (see [4]).

What is the relationship of these $\mathcal{ALC}$ extensions to qualitative spatial reasoning? In the field, the so-called RCC-family of spatial reasoning calculi is well-known (see [3]). A description of a concrete spatial scene with RCC-relationships can be seen as a complete edge-colored graph — between any two spatial objects, *exactly one* so-called base relation holds. This is called the *JEPD-property* (jointly exhaustive and pairwise disjoint). In the case of RCC8, we can distinguish eight, and in the case of RCC5 only five base relations – some qualitative distinctions are ignored, see Figure 1. It even makes sense to define RCC3, RCC2, and RCC1, offering coarser and coarser spatial description capabilities. We admit that the main purpose for us to consider them here is to get a better understanding of the problems involved.

Given that we do not have complete definite knowledge concerning a scene (some spatial relationships might be unknown or only vaguely known), we can combine general logical inferences with spatial inferences in order to either infer more specific spatial relationships, or to get more appropriate descriptions of the objects involved in that scene (see also [2]). For example, if the relationship between object a and c is not specified (in fact, this corresponds to the disjunction of all base relations) but given that we know the relation between a and another object b (say, S) and also between b and c (e.g. T), then we can read off the possible base relations between a and c from the entry in the so-called *RCC composition table* for $S \circ T$ (see Figure 2a). Each entry represents an inference pattern of the form $\forall x, y, z : S(x, y) \wedge T(y, z) \Rightarrow (R_1(x, z) \vee \ldots \vee R_n(x, z))$, which is translated into a corresponding $\mathcal{ALC}_{\mathcal{RA}}$ role axiom $S \circ T \sqsubseteq R_1 \sqcup \cdots \sqcup R_n$. Corresponding to the underlying composition table, we call these special $\mathcal{ALC}_{\mathcal{RA}}$ specializations $\mathcal{ALC}_{\mathcal{RCC}8}$, $\mathcal{ALC}_{\mathcal{RCC}5}$, and so on. If we handle these relations as roles in $\mathcal{ALC}_{\mathcal{RA}}$, we can use universal quantification (concepts of the form $\forall R.D$) to pose additional constraints on c. It is our conviction that the ability to *quantify over roles corresponding to spatial relationships* is a key-ability and first-order requirement for the qualitative spatial reasoning applications that we are trying to realize.[1] However, the class of concepts that might be used as qualifications within universal quantifiers is subject to discussion. It is obvious that quantification involving only pure *propositional concepts* is much easier to handle than full quantification. It should be noted that there are various (space and time) logics that do not offer universal quantification at all.

However, $\mathcal{ALC}_{\mathcal{RA}}$— with arbitrary role boxes — is undecidable, and the qualitative spatial reasoning specialization $\mathcal{ALC}_{\mathcal{RCC}}$ only make sense if also the appropriate *inverse relationships* of the base relations are respected. It is of course important that the inverse relationship must be respected, since some spatial inferences cannot be drawn otherwise. For example, the concept $(\exists DR.(C \sqcap \forall DR.C)) \sqcap \exists PPI.\neg C$ is unsatisfiable in $\mathcal{ALCI}_{\mathcal{RCC}5}$, but satisfiable in $\mathcal{ALC}_{\mathcal{RCC}5}$. This is due to the fact that $DR \circ PPI \rightarrow DR$, and therefore, the two $\exists$-successors are linked via DR in the case of $\mathcal{ALCI}_{\mathcal{RCC}5}$ due to the requirement that $DR^{\mathcal{I}} = (DR^{\mathcal{I}})^{-1}$ which is only respected in $\mathcal{ALCI}_{\mathcal{RCC}5}$, but not in $\mathcal{ALC}_{\mathcal{RCC}5}$. Concerning $\mathcal{ALCI}_{\mathcal{RCC}8}$, the question whether it might be decidable or not was already raised by Cohn in (slightly different form in) [1], where he suggests to use a pair of modal operators $\Box_R$ and $\Diamond_R$ for each available spatial base relationship R of RCC. Subsequent work with Bennett then focused on encoding of RCC relationships and networks in modal and intuitionistic (propositional) logic. However, they did not investigate quantification over RCC relationships.

[1]Currently, only the logic $\mathcal{ALCRP}(\mathcal{S}_2)$ offers this ability (see [2]). Unfortunately, $\mathcal{ALCRP}(\mathcal{S}_2)$ somehow suffers from a severe restriction regarding the allowed quantifier patterns in order to achieve decidability.

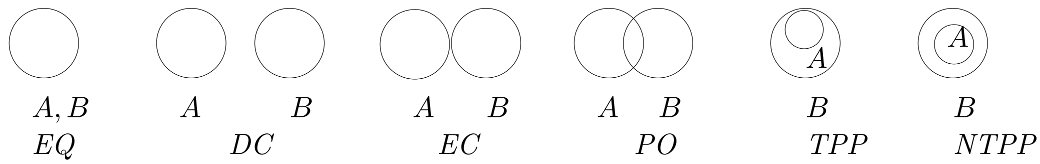


Figure 1: RCC8 Relationships, f.l.t.r.: Equal, Disconnected, Externally Connected, Partial Overlap, Tangential Proper Part, Non-Tangential Proper Part. To be read as $EQ(A, B)$, etc. TPP and $NTPP$ have inverses: $TPPI$ and $NTPPI$. In RCC5, the relationships DC and EC are collapsed into DR; TPP and $NTPP$ into PP, as well as $TPPI$ and $NTPPI$ into PPI. In RCC3, $\{PP, PPI, PO\}$ are collapsed into ONE ("Overlapping but not equal"), and ONE, EQ into O in the case of RCC2 ("Overlapping"). The coarsest version – RCC1 – has only one relationship, called "Spatially Related", SR.

In contrast to our previous work we are now considering concept satisfiability in the $\mathcal{ALCI}_{\mathcal{RCC}}$ logics, and we investigate whether respecting the inverse relationships makes the satisfiability problem in these logics easier (contrasted with the problems we had showing decidability or undecidability of these logics *without* the corresponding inverse relationships, e.g. the $\mathcal{ALC}_{\mathcal{RCC}}$ logics). But why should this be the case? Mainly due to the fact that most roles of RCC are symmetric, and that the considered logics seem to be relatives of various "S5"-like logics. In $\mathcal{ALCI}_{\mathcal{RCC}5}$, the only non-symmetric roles are "proper part" (PP) and its inverse (PPI), in $\mathcal{ALCI}_{\mathcal{RCC}8}$ we have "non-tangential proper part" (NTPP) and "tangential proper part" (TPP) and their corresponding inverses (NTPPI and NTPP) - in fact, these are even asymmetric (and therefore irreflexive). Due to the fact that the composition of two arbitrary roles is always defined, models of $\mathcal{ALCI}_{\mathcal{RCC}}$ are in fact complete graphs with roles being interpreted as undirected edges in this complete graph (with the exception of the asymmetric roles). The nodes are labeled with propositional information, as in $\mathcal{ALC}$. Each node is linked to every other node in the model. The models might be called "S5"-like models, due to the modal logic "S5', which is the normal modal logic of transitive, reflexive and symmetric frames (more specifically, of an equivalence relation).

2 Some Observations & Questions

More formally, the family of $\mathcal{ALCI}_{\mathcal{RCC}}$ logics is defined as follows:

Definition 1 (Syntax and Semantics of $\mathcal{ALCI}_{\mathcal{RCC}}$) Each $\mathcal{ALCI}$ concept is a valid $\mathcal{ALCI}_{\mathcal{RCC}}$ concept: every concept name is a concept, and if R is a role from $\mathcal{N}_{\mathcal{R}}$, and C and D are concepts, then also $\neg C$, $C \sqcap D$, $C \sqcup D$, $\exists R.C$ and $\forall R.C$ are concepts. In the following, disjunctions of (base) relations are written in curly brackets. If $R = \{S_1, \ldots, S_n\}$ is a disjunction of (base) relations S_i, we also write $\forall\{S_1, \ldots, S_n\}.C$ ($\exists\{S_1, \ldots, S_n\}.C$) as a shorthand for $\forall S_1.C \sqcap \ldots \sqcap \forall S_n.C$ ($\exists S_1.C \sqcup \ldots \sqcup \exists S_n.C$). In the case of $\mathcal{ALCI}_{\mathcal{RCC}8}$ the set of role names $\mathcal{N}_{\mathcal{R}}$ is $\mathcal{N}_{\mathcal{R}} = \{DC, EC, PO, EQ, TPP, TPPI, NTPP, NTPPI\}$; in the case of $\mathcal{ALCI}_{\mathcal{RCC}5}$ we have $\mathcal{N}_{\mathcal{R}} = \{DR, PO, EQ, PP, PPI\}$, where $PP =_{def} \{TPP, NTPP\}$ and $PPI =_{def} \{TPPI, NTPPI\}$; $\mathcal{N}_{\mathcal{R}} = \{DR, ONE, EQ\}$ in the case of $\mathcal{ALCI}_{\mathcal{RCC}3}$, where $ONE =_{def} \{PP, PPI, PO\}$ ("ONE" for "overlapping but not equal"); $\mathcal{N}_{\mathcal{R}} = \{DR, O\}$ with $O =_{def} \{ONE, EQ\}$ in the case of $\mathcal{ALCI}_{\mathcal{RCC}2}$, and finally, $\mathcal{ALCI}_{\mathcal{RCC}1}$ with $\mathcal{N}_{\mathcal{R}} = \{SR\}$, where $SR =_{def} \{O, DR\}$ ("SR" for "spatially related").

We use the function inv to refer to the corresponding converse role (e.g. $PPI = \mathsf{inv}(PP)$, $DR = \mathsf{inv}(DR)$). Please note that inv is total on $\mathcal{N_R}$.

A model of a concept is an *interpretation* $\mathcal{I} =_{def} (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ that is a usual $\mathcal{ALC}$ model for that concept, i.e. maps concept names to subsets of $\Delta^{\mathcal{I}}$, roles to subsets of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, but additionally satisfies the disjointness, converse and composition requirements as specified by the corresponding RCC composition table. That is, for all roles $R, S \in \mathcal{N_R}$, $R \neq S$ we have $R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$, and if $S \circ T \sqsubseteq R_1 \sqcup \cdots \sqcup R_n$ is an entry in the RCC composition table, then $S^{\mathcal{I}} \circ T^{\mathcal{I}} \subseteq R_1^{\mathcal{I}} \cup \cdots \cup R_n^{\mathcal{I}}$ holds, and if $R = \mathsf{inv}(S)$, then $R^{\mathcal{I}} = (\mathsf{inv}(S)^{\mathcal{I}})^{-1}$. Please note that we are working with a purely abstract semantics - we do not give a specific truly spatial interpretation for the objects as two-dimensional regions or the like here.

Obviously, SR has to be an equivalence relation – every object is spatially related to every other object (including itself). In the case of RCC2 we additionally require that $\{ <x, x> \mid x \in \Delta^{\mathcal{I}} \} \subseteq O^{\mathcal{I}}$, and in the case of RCC3, RCC5 and RCC8 $\{ <x, x> \mid x \in \Delta^{\mathcal{I}} \} \subseteq EQ^{\mathcal{I}}$ must hold. This is plausible if we think of EQ as equality — it might even be plausible to require that the interpretation of EQ is *exactly* the identity relation: $EQ^{\mathcal{I}} = \{ <x, x> \mid x \in \Delta^{\mathcal{I}} \}$. We call this *the strong(er) EQ semantics* (or the *weak(er) EQ semantics*, respectively). The stronger EQ semantics is appealing from a practical point of view since it makes the plausible assumption that no two different congruent objects in the world might exist.

As usually, we say that an interpretation $\mathcal{I}$ is a model of a concept C, written $\mathcal{I} \models C$, if and only if $C^{\mathcal{I}} \neq \emptyset$. □

In the following we make some observations concerning the considered DLs. First of all, none hast the tree model property, but we claim that some have the finite model property (see below).

$\mathcal{ALCI}_{\mathcal{RCC}1}$ is decidable and equivalent to the modal logic "S5". It is well-known that "S5" is NP-complete. We refer to the nesting depth of $\Box$ and $\Diamond$ modalities as *modal degree.* From modal logics we know that every "S5" formula having a modal degree higher than one can be reduced to an equivalent "S5" formula having degree one. This is due to the equivalences $\Diamond p \equiv \Box\Diamond p$ (equivalently, $\exists SR.C \Leftrightarrow \forall SR.\exists SR.C$ for all concepts C), $\Box p \equiv \Diamond\Box p$, $\Diamond p \equiv \Diamond\Diamond p$, $\Box p \equiv \Box\Box p$ that are valid in "S5", and which allow us to discard all nested modalities but the last one in an "S5" formula. Each "S5" formula can therefore be brought into *modal conjunctive normal form,* where each conjunct is a disjunction of the form $\beta \vee \Box\gamma_1 \vee \ldots \vee \Box\gamma_n \vee \Diamond\delta_1 \vee \ldots \vee \Diamond\delta_m$, such that all β, δ_i and γ_j are propositional formulas.

$\mathcal{ALCI}_{\mathcal{RCC}2}$ is decidable as well. The composition table is trivial: $\{DR \circ O \rightarrow \{DR, O\}, DR \circ DR \rightarrow \{DR, O\}, O \circ O \rightarrow \{DR, O\}\}$. It is obvious that every complete and $\{DR, O\}$-colored graph satisfies the role box axioms. The validities (axiom schemas) of this logic include the axioms $\exists O.C \Rightarrow \forall O.(C \sqcup \exists\{O, DR\}.C) \sqcap \forall DR.\exists\{O, DR\}.C)$, $\exists DR.C \Rightarrow \forall DR.(C \sqcup \exists\{O, DR\}.C) \sqcap \forall O.\exists\{O, DR\}.C$, $\forall O.C \Rightarrow C$. It is open whether $\mathcal{ALCI}_{\mathcal{RCC}2}$ is still NP-complete. In fact, it seems to be impossible to enforce exponentially large models. We therefore conjecture it has the polysize-model property.

In the case of $\mathcal{ALCI}_{\mathcal{RCC}3}$ we have to distinguish between $\mathcal{ALCI}_{\mathcal{RCC}3}$ with the strong EQ semantics and the weak EQ semantics: for example, $\exists EQ.(C \sqcap \exists EQ.\neg C)$ is satisfiable only under the weak EQ semantics, but unsatisfiable otherwise. The composition table is as expected (without symmetric entries): $\{DR \circ ONE \rightarrow \{DR, ONE\}, DR \circ DR \rightarrow \{DR, ONE, EQ\}, ONE \circ ONE \rightarrow \{DR, ONE, EQ\}, EQ \circ DR \rightarrow \{DR\}, EQ \circ ONE \rightarrow \{ONE\}, EQ \circ EQ \rightarrow \{EQ\}\}$. In order to prove decidability of $\mathcal{ALCI}_{\mathcal{RCC}3}$, we now give a reduction from concept satisfiability in $\mathcal{ALCI}_{\mathcal{RCC}3}$ to satisfiability in first order predicate logic with two variables *and equality*, which is a decidable logic.

Definition 2 Let C be an $\mathcal{ALCI}_{\mathcal{RCC}3}$ concept in negation normal norm (NNF). Moreover, we assume that each concept C occurring within $\exists R.C$ and $\forall R.C$ is in disjunctive normal form (DNF), such that each conjunct in the disjunction of conjunctions is either an atomic concept, a negated atomic concept, or a concept of the form $\exists S.D$ or $\forall S.D$, where D is again in DNF and NFF, etc. We then assume that there is a function α, which, applied to a disjunct D of the above DNF (note that D is itself a conjunction), returns the *modal part of* D, and that there is a corresponding function β which returns the *propositional part of* D, e.g. if $D = A_1 \sqcap (\neg A_2) \sqcap \exists R.E \sqcap \forall S.F$, then $\alpha(D) = \{\exists R.E, \forall S.F\}$ and $\beta(D) = \{A_1, (\neg A_2)\}$. We skip the (easy) definitions of α and β here, as well as for DNF and NNF. The following two mutually recursive functions ϕ_x and ϕ_y do the main job (ϕ_y is obtained from ϕ_x by swapping x and y):

$$
\begin{aligned}
&\phi_x(C) =_{def} C(x), \text{ if } C \text{ is an atomic concept}\\
&\phi_x(\neg C) =_{def} \neg\phi_x(C)\\
&\phi_x(C_1 \sqcap \ldots \sqcap C_n) =_{def} \phi_x(C_1) \wedge \ldots \wedge \phi_x(C_n)\\
&\phi_x(C_1 \sqcup \ldots \sqcup C_n) =_{def} \phi_x(C_1) \vee \ldots \vee \phi_x(C_n)\\
&\phi_x(\exists EQ.C) =_{def} \quad (\textstyle\bigwedge_{mp \in \alpha(C)} \phi_x(mp) \wedge\\
&\qquad\qquad\qquad\qquad (\exists y : EQ'(x,y) \wedge EQN(y) \wedge \textstyle\bigwedge_{bp \in \beta(C)} \phi_y(bp))\\
&\qquad\qquad\qquad\qquad \vee\phi_x(C)), \text{ if } C \text{ is not a disjunction}\\
&\phi_x(\exists EQ.(C_1 \sqcup \ldots \sqcup C_n)) =_{def} \phi_x(\exists EQ.C_1) \vee \ldots \vee \phi_x(\exists EQ.C_n)\\
&\phi_x(\exists R.C) =_{def} (\exists y : R(x,y) \wedge \phi_y(\exists EQ.C)), \text{ if } R \neq EQ\\
&\phi_x(\forall EQ.C) =_{def} \quad (\textstyle\bigwedge_{mp \in \alpha(C)} \phi_x(mp) \wedge\\
&\qquad\qquad\qquad\qquad (\forall y : EQ'(x,y) \wedge EQN(y) \Rightarrow \textstyle\bigwedge_{bp \in \beta(C)} \phi_y(bp))\\
&\qquad\qquad\qquad\qquad \wedge\phi_x(C)), \text{ if } C \text{ is not a disjunction}\\
&\phi_x(\forall EQ.(C_1 \sqcup \ldots \sqcup C_n)) =_{def} \neg(\phi_x(\exists EQ.\mathsf{DNF}(\mathsf{NNF}((\neg C_1 \sqcap \ldots \sqcap \neg C_n)))))\\
&\phi_x(\forall R.C) =_{def} (\forall y : R(x,y) \Rightarrow \phi_y(\forall EQ.C)), \text{ if } R \neq EQ
\end{aligned}
$$

The "FO2=" translation of C is then defined as follows (note the use of "="):

$$
\begin{aligned}
&\forall x, y : DR(x,y) \oplus ONE(x,y) \oplus x = y \oplus (EQN(x) \vee EQN(y)) \wedge\\
&\forall x, y : DR(x,y) \Leftrightarrow DR(y,x) \wedge\\
&\forall x, y : ONE(x,y) \Leftrightarrow ONE(y,x) \wedge\\
&\phi_x(C)
\end{aligned}
\qquad \square
$$

We claim that the resulting formula is equi-satisfiable with C under the weak EQ semantics in $\mathcal{ALCI}_{\mathcal{RCC}3}$. The proof can be found in a forthcoming report.

It is obvious that already FO2 *without* equality is sufficient if one translates $\mathcal{ALCI}_{\mathcal{RCC}3}$ with the *strong* EQ semantics into FOPL. But considering the weak EQ semantics, we have to ensure that $\forall x, y, z : EQ(x, z) \Leftrightarrow DR(x, y) \wedge DR(y, z) \oplus ONE(x, y) \wedge ONE(y, z) \oplus EQ(x, y) \wedge EQ(y, z)$ is satisfied. In order to achieve an equi-satisfiable FO2 formula (using only two variables!) we have to exploit the special properties of "=", which ensures that equal objects are in fact interpreted as identical domain objects. The trick is that "$x = y$" is used to represent $EQ(x, y)$. Whenever two objects are forced to be EQ, the "=" relationship ensures that the RCC network cannot become inconsistent without being recognized as inconsistent. Suppose that $EQ(x, y)$, and that for some z, we have $DR(x, z)$ and $ONE(z, y)$. The network is obviously inconsistent. Fortunately, due to "x=y", representing $EQ(x, y)$, the inconsistency will be detected: $DR(x, z)$ implies $DR(y, z)$ and $DR(z, y)$ due to symmetry, together with $ONE(z, y)$ violating the disjointness requirement. However, note that under the weak EQ semantics, x and y could very well have different propositional descriptions in $\mathcal{ALCI}_{\mathcal{RCC}3}$, even if $EQ(x, y)$ holds. Thus, one has to *separate the modal and the propositional point of view of the "EQ"-connected objects.* For example, $\exists EQ.C \sqcap \exists EQ.\neg C$ is *consistent*, but translating this into $\exists x, y, z : x = y \wedge C(y) \wedge y = z \wedge \neg C(z)$ obviously yields an unsatisfiable formula. The separation is therefore achieved by using a further binary predicate "EQ'". Nested occurrences of $\exists EQ. \ldots$ and $\forall EQ. \ldots$-concepts are flattened during the translation, similar to the "S5" modal conjunctive normal form which has a modal degree of one (see above).

It should be noted that the difference between $\mathcal{ALCI}_{\mathcal{RCC}3}$ with the strong and the weak EQ semantics is that the former requires that $EQ^{\mathcal{I}}$ is a congruence relation for all present unary and binary predicates (that is, concepts and roles), whereas the latter only enforces that $EQ^{\mathcal{I}}$ is a congruence relation for all present binary predicates. The weak EQ semantics can, of course, be made "strong" by conjoining, for all relevant concept names C, the global axioms $C \rightarrow \forall EQ.C$ to the original concept (add $\forall \{EQ, DR, ONE\}.(C \rightarrow \forall EQ.C)$ as a conjunct to the original concept). Then, it is easy to see that all EQ-connected nodes in a model (they form an EQ-clique) can be collapsed into a single node, and we still have a model.

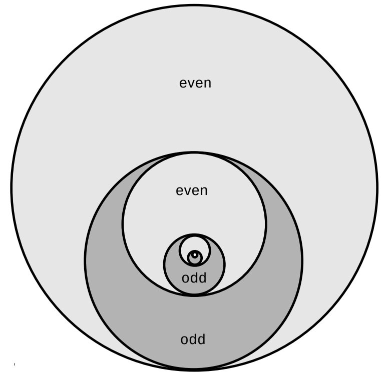
Considering $\mathcal{ALCI}_{\mathcal{RCC}5}$ and $\mathcal{ALCI}_{\mathcal{RCC}8}$ we can observe that neither has the finite model property, unlike $\mathcal{ALCI}_{\mathcal{RCC}3}$ and its sub-languages. Due to the asymmetric and transitive PP relation in $\mathcal{ALCI}_{\mathcal{RCC}5}$, for example, the concept $((\exists PP.\top) \sqcap (\forall PP.\exists PP.\top))$ has no finite models. See below for a short discussion.

There is some indication that $\mathcal{ALCI}_{\mathcal{RCC}5}$ could possibly be computationally easier to handle than $\mathcal{ALCI}_{\mathcal{RCC}8}$, since the latter seems to have more expressive power. More specifically, unlike $\mathcal{ALCI}_{\mathcal{RCC}5}$, $\mathcal{ALCI}_{\mathcal{RCC}8}$ somehow allows the distinction of a role and its *transitive orbit* (this is a role whose interpretation contains at least the transitive closure of the interpretation of the generating role; "somehow" because it is not strictly a transitive orbit, see below).

The following concept enforces an infinite chain of *even*-*odd*-…-marked individuals, see the "spatial illustrations" in Figure 2b. Each node can distinguish its direct *TPPI*-successor from all its indirect *NTPPI*-successors. Laxly speaking, we can consider *NTPPI* somehow as the transitive orbit of *TPPI*; more specifically, we have $((TPPI^{\mathcal{I}})^{+} - TPPI^{\mathcal{I}}) \subseteq NTPPI^{\mathcal{I}}$ (please assume that $odd =_{def} \neg even$):

∘	DR(a,b)	PO(a,b)	EQ(a,b)	PPI(a,b)	PP(a,b)
DR(b,c)	T	DR PO PPI	DR	DR PO PPI	DR
PO(b,c)	DR PO PP	T	PO	PO PPI	DR PO PP
EQ(b,c)	DR	PO	EQ	PPI	PP
PP(b,c)	DR PO PP	PO PP	PP	PO EQ PP PPI	PP
PPI(b,c)	DR	DR PO PPI	PPI	PPI	T

a) RCC5 Composition Table;
$T =_{def} \{DR, PO, EQ, PP, PPI\}$



b) Illustration of an infinite $\mathcal{ALCI}_{\mathcal{RCC}8}$ model of *even_odd_chain*

Figure 2: RCC5 table and infinitely descending model

$$
\begin{aligned}
\mathit{even\_odd\_chain} =_{def}\ & \mathit{even} \sqcap \\
& (\exists TPPI.\exists TPPI.\top) \sqcap \\
& (\mathit{even} \Rightarrow \forall TPPI.\mathit{odd}) \sqcap \\
& (\mathit{odd} \Rightarrow \forall TPPI.\mathit{even}) \sqcap \\
& (\forall NTPPI.((\mathit{even} \Rightarrow \forall TPPI.\mathit{odd}) \sqcap (\mathit{odd} \Rightarrow \forall TPPI.\mathit{even}))) \sqcap \\
& (\forall TPPI.((\mathit{even} \Rightarrow \forall TPPI.\mathit{odd}) \sqcap (\mathit{odd} \Rightarrow \forall TPPI.\mathit{even}))) \sqcap \\
& (\forall NTPPI.\exists TPPI.\top)
\end{aligned}
$$

Due to the RCC8 composition table, we have

$$TPPI \circ TPPI \rightarrow \{TPPI, NTPPI\}, TPPI \circ NTPPI \rightarrow \{NTPPI\},$$
$$NTPPI \circ TPPI \rightarrow \{NTPPI\}, NTPPI \circ NTPPI \rightarrow \{NTPPI\}.$$

This possibility came to us as a surprise. Once a role and its transitive orbit (or closure) can be distinguished, it is easy to see that even finite pseudo-models representing infinite $\mathcal{ALCI}_{\mathcal{RCC}8}$ models would have to be exponential in size of the length of the input concept. Finite pseudo-models are, for example, constructed by tableau calculi for logics like $\mathcal{SHIQ}$ that also lack the finite model property and whose infinite models can be represented finitely by means of a so-called blocked tableau, which can be understood as a pseudo-model. We can observe that a calculus for $\mathcal{ALCI}_{\mathcal{RCC}8}$ would at least have to construct pseudo-models of exponential size in the length of the input concept. This is demonstrated with the following classical "binary n-bit counter" concept:

$$
\begin{aligned}
\mathit{counter} =_{def}\ & \mathit{even\_odd\_chain} \sqcap \\
& \exists TPPI.\exists TPPI.(\neg bit_0 \sqcap \neg bit_1 \sqcap \ldots \sqcap \neg bit_{n-1}) \sqcap \\
& \forall NTPPI.(\mathit{toggle\_bit}_0 \sqcap \mathit{toggle\_bit}_1 \sqcap \ldots \sqcap \mathit{toggle\_bit}_{n-1}) \\
\mathit{toggle\_bit}_0 =_{def}\ & (bit_0 \sqcap \forall TPPI.\neg bit_0) \sqcup (\neg bit_0 \sqcap \forall TPPI.bit_0) \\
\mathit{toggle\_bit}_i =_{def}\ & ((\sqcap_{0 \le j < i} bit_j) \sqcap ((bit_i \sqcap \forall TPPI.\neg bit_i) \sqcup (\neg bit_i \sqcap \forall TPPI.bit_i))) \sqcup \\
& (\neg(\sqcap_{0 \le j < i} bit_j) \sqcap ((bit_i \sqcap \forall TPPI.bit_i) \sqcup (\neg bit_i \sqcap \forall TPPI.\neg bit_i)))
\end{aligned}
$$

There does not seem to be a way to achieve a similar effect in $\mathcal{ALCI}_{\mathcal{RCC}5}$. Please note that each pseudo-model would most likely have to represent an exponential number of nodes (e.g. $2 + 2^n$ here), since even propositional clashes could not be detected otherwise — say, the conjunction $\sqcap_{0 \le j < n} bit_j$ turns out to be unsatisfiable which can only be detected in the $2 + 2^n$th node.

3 Future Work - Finite Model Reasoning with $\mathcal{ALCI}_{\mathcal{RCC}5}$?

If we have such problems showing decidability or undecidability of $\mathcal{ALCI}_{\mathcal{RCC}5}$ and $\mathcal{ALCI}_{\mathcal{RCC}8}$, then might it probably be easier to restrict ourselves to *finite model reasoning*, i.e. impose a semantics where concepts that have only infinite models are considered as *unsatisfiable?* This would also be appealing from an application point of view. Generally speaking, finite model reasoning is not necessarily easier than reasoning with general models (Trakhtenbrot). However, it *may be easier*. For example, $(\exists PP.\top) \sqcap (\forall PP.\exists PP.\top)$, $(\exists DR.\top) \sqcap (\forall DR.\exists PPI.\top)$, and $(\exists DR.\exists PO.C) \sqcap (\forall PO.\neg C) \sqcap (\forall DR.\neg C) \sqcap (\forall PP.((\exists DR.\exists PO.C) \sqcap (\forall PO.\neg C) \sqcap (\forall DR.\neg C)))$ only have infinite models. It is tempting to suspect that "'PP" and/or "PPI" within universal and/or existential value restrictions are responsible for spawning the infinite models.

A finite model reasoning $\mathcal{ALCI}_{\mathcal{RCC}5}$ calculus would have to detect the unsatisfiability of all given examples. We were experimenting with some kind of "infinity checker" that tried to detect whenever an infinite structure is enforced by the current tableau expansion history (of course, this might be undecidable as well). But note that an "infinity checker" is in fact a *stronger* predicate than a blocking condition — the blocking condition would have to ensure that whenever it returns TRUE, an infinite *model* can be constructed. In contrast, the infinity checker must not know whether the concept under consideration is satisfiable in the infinite or not. The problem we are trying to solve seems to be related to the well-foundedness problem in *part-whole-reasoning.*

Summing up, we have made some first steps from the general $\mathcal{ALC}_{\mathcal{RA}^\ominus}$, $\mathcal{ALC}_{\mathcal{RA}}$ and $\mathcal{ALC}_{\mathcal{RASG}}$ logics, missing inverse roles, to logics that are useful for qualitative spatial reasoning. We are optimistic that at least for $\mathcal{ALCI}_{\mathcal{RCC}5}$ the mentioned problems can be overcome. It is also easy to see that there are restricted versions of $\mathcal{ALCI}_{\mathcal{RCC}5}$ and $\mathcal{ALCI}_{\mathcal{RCC}8}$ whose decidability can easily be shown; e.g. allow only propositional concepts in universal quantifiers, etc. But this is not our mission.

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