On Terminological Default Reasoning about Spatial Information

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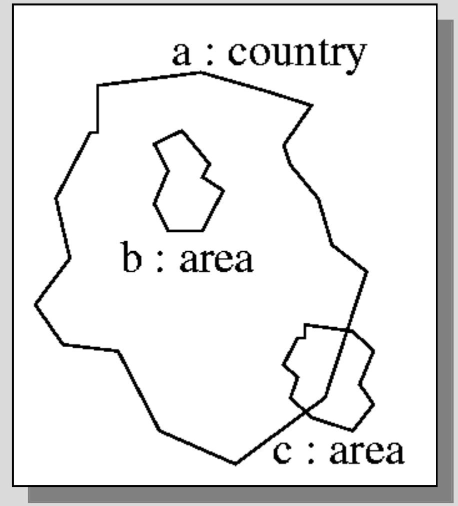
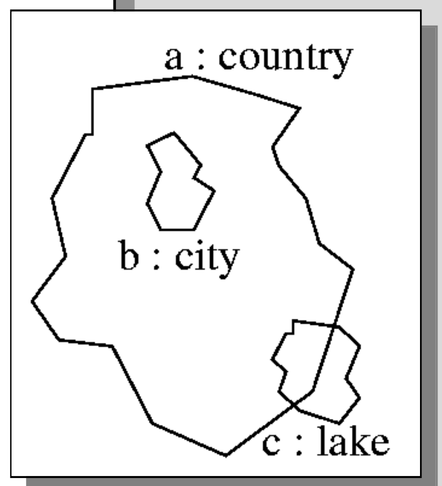
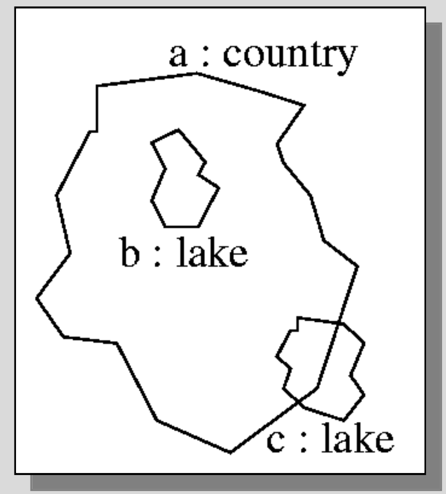
- ❑ Motivation
- ❑ Preliminaries
  - ➪ spatioterminological reasoning with $\mathcal{ALCRP}(\mathcal{S}_2)$ [Lutz, Haarslev & Möller]
  - ➪ nonmonotonic reasoning with Reiter´s default logic
    - terminological default rules / theories [Baader & Hollunder]
- ❑ Spatioterminological default reasoning
  - ➪ on computing extensions
- ❑ Conclusion & future work

Motivation: Incomplete Spatioterminological Knowledge

- Combination of terminological, spatial & default reasoning techniques
  - Geographic Information Systems (GIS)
- Terminological knowledge
  - **capital_city . city**
- Spatial knowledge
  - properties of spatial relationships, eg. **tpp** (contains) is **transitive**
- Spatioterminological knowledge
  - a **city** is **contained** within exactly one **country**
  - two **countries** never **overlap** each other
- Default knowledge
  - data augmentation / completion
  - „b“ could possibly be a **city** or a **lake**, but not both (disjoint concepts)
  - ABox realization would not work



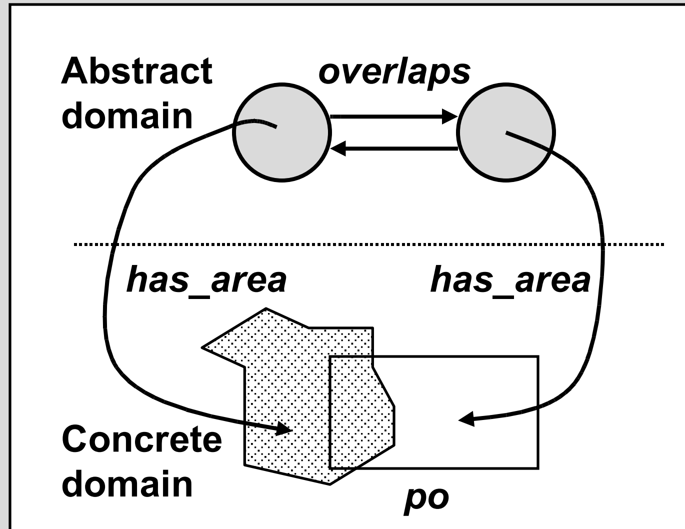




Spatioterminological Reasoning with $\mathcal{ALCRP}(\mathcal{S}_2)$

- $\mathcal{ALCRP}(\mathcal{D})$ extends $\mathcal{ALC}(\mathcal{D})$
  - $\mathcal{ALCRP}(\mathcal{D})$ = $\mathcal{ALC}(\mathcal{D})$+ role-forming predicate-based operator
  - decidable for restricted concept terms
  - restrictedness closed under negation
- $\mathcal{ALCRP}(\mathcal{S}_2)$
  - admissible concept domain S_2, regular closed subsets of $\Re^2$, called **regions**, with **RCC8 predicates**
  - properties of relationships captured by concrete domain, e.g. transitivity of **tpp**
  - RCC8 predicates
    - **dc, ec, po, tpp, ntpp, tppi, ntppi, eq**
  - defined roles, TBox axioms

$$
\begin{aligned}
inside &\doteq \exists(has\_area)(has\_area).\textbf{tpp-ntpp}\\
contains &\doteq \exists(has\_area)(has\_area).\textbf{tppi-ntppi}\\
overlaps &\doteq \exists(has\_area)(has\_area).\textbf{po}
\end{aligned}
$$



$$
\begin{aligned}
&(\exists(u_1,\ldots,u_n)(v_1,\ldots,v_m).P)^{\mathcal{I}} := \\
&\quad \{(a,b)\in\Delta_{\mathcal{I}}\times\Delta_{\mathcal{I}} \mid \\
&\qquad \exists x_1,\ldots,x_n,y_1,\ldots,y_m\in\Delta_{\mathcal{D}}: \\
&\qquad\quad (a,x_1)\in u_1^{\mathcal{I}},\ldots,(a,x_n)\in u_n^{\mathcal{I}}, \\
&\qquad\quad (b,y_1)\in v_1^{\mathcal{I}},\ldots,(b,x_m)\in v_m^{\mathcal{I}}, \\
&\qquad\quad (x_1,\ldots,x_n,y_1,\ldots,y_m)\in P^{\mathcal{D}}\}
\end{aligned}
$$

Spatioterminological Background Knowledge (TBox)

$$
\begin{aligned}
area &\doteq \exists has\_area.\textbf{is-region} \\
country\_region &\dot{\sqsubseteq} \neg natural\_region \sqcap \\
&\quad large\_scale \sqcap area \\
city\_region &\dot{\sqsubseteq} \neg natural\_region \sqcap \\
&\quad \neg large\_scale \sqcap area \\
lake\_region &\dot{\sqsubseteq} natural\_region \sqcap area \\
country &\doteq country\_region \sqcap \\
&\quad \forall contains.\neg country\_region \sqcap \\
&\quad \forall overlaps.\neg country\_region \sqcap \\
&\quad \forall inside.\neg country\_region \\
city &\doteq city\_region \sqcap \\
&\quad \exists inside.country\_region \\
lake &\dot{\sqsubseteq} lake\_region
\end{aligned}
$$

Default Theories & Terminological Default Theories

- Default rules [Reiter, 1980]
  - α prerequisite, β_i justifications, γ conclusion, FOPL formulae
- Default theory (*W,D*)
  - *W* = world description
  - *D* = set of defaults
- Different sets of **extensions** of (*W,D*)
  - **sceptical** vs. **credulous** consequence
- **Terminological** default theories [Baader & Hollunder, 1991]
  - α, β_i, γ concept terms
  - *W* = ABox, *D* = set of closed default rules
  - restricted semantics, no skolemization
  - consequence problem decidable
- Closing concept terms over ABox *W*
  - concept terms => ABox concept membership assertions

$$\frac{\alpha : \beta_1, \beta_2, \ldots, \beta_n}{\gamma}$$

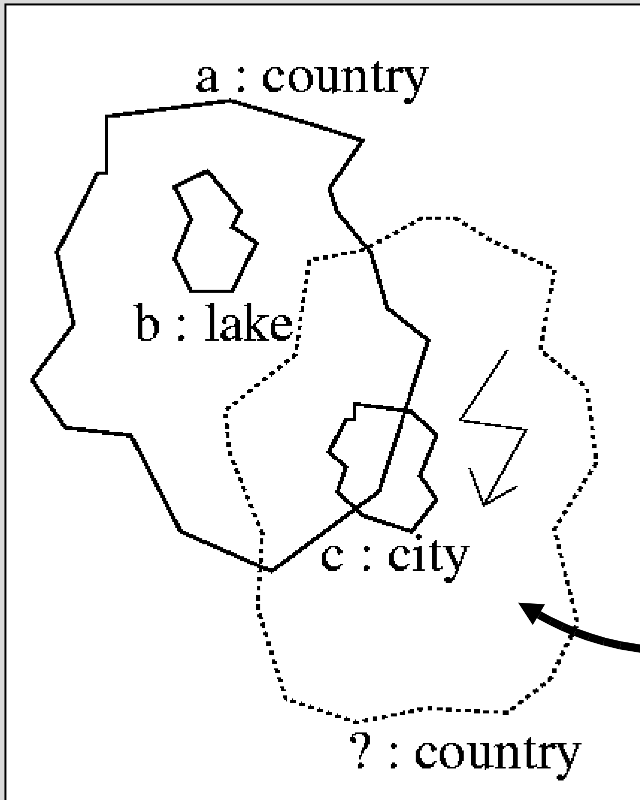
Example

$$\frac{area : country}{country}$$

$$\frac{area : city}{city}$$

$$\frac{area : lake}{lake}$$

Closed Defaults



$$\frac{\{a: area\}:\{a: country\}}{\{a: country\}} \qquad \frac{\{a: area\}:\{a: city\}}{\{a: city\}} \qquad \frac{\{a: area\}:\{a: lake\}}{\{a: lake\}}$$

„XOR“

$$\frac{\{b: area\}:\{b: country\}}{\{b: country\}} \qquad \frac{\{b: area\}:\{b: city\}}{\{b: city\}} \qquad \frac{\{b: area\}:\{b: lake\}}{\{b: lake\}}$$

$$\frac{\{c: area\}:\{c: country\}}{\{c: country\}} \qquad \frac{\{c: area\}:\{c: city\}}{\{c: city\}} \qquad \frac{\{c: area\}:\{c: lake\}}{\{c: lake\}}$$

- World description *W* =
  { *a* : *country, b* : *area, c* : *area,*
  (*a,b*) : *contains,* (*b,a*) : *inside,*
  (*a,c*) : *overlaps,* (*c,a*) : *overlaps* }

- Closing over *W* yields 9 closed defaults
- Two extensions
  - ***E1* = *W* U { <u>*b* : *city*</u>, *c* : *lake* }**
  - ***E2* = *W* U { <u>*b* : *lake*</u>, *c* : *lake* }**
- ***W* U { *b* : *lake, c* : *city* }** inconsistent, see picture
- 2 sets of „generating defaults“

Spatioterminological Default Theories with „ABox Patterns“

- We also want to conclude complex role assertions
  - ***W* = { *linköping* : *swedish_city, sweden* : *country* }**
  - ***E* = *W* U { (*sweden, linköping*) : *contains* }**
  - cannot be expressed with concept terms as α, β_i, γ
- „ABox patterns“
  - ABoxes with **variables**, e.g. *X, Y, Z*
  - to be closed over *W*
  - can also refer to specific ABox individuals
- „Duality“
  - use known concept memberships to conclude spatial relationships
  - use spatial relationships to deduce concept memberships

$$\frac{\{X : swedish_{city}, sweden : country\} : \{(sweden, X) : contains\}}{\{(sweden, X) : contains\}}$$

Close

$$\frac{\{linköping : swedish_{city}, sweden : country\} : \{(sweden, linköping) : contains\}}{\{(sweden, linköping) : contains\}}$$

On Computing Extensions

Let E be a set of closed formulae and (A, D) be a closed default theory. We define $E_0 := A$ and for all $i \geq 0$

$$E_{i+1} := E_i \cup \{\gamma \mid \alpha : \beta_1, \ldots, \beta_n / \gamma \in D, \; \alpha \in Th(E_i), \; \neg\beta_1, \ldots, \neg\beta_n \notin Th(E)\}.$$

Then, $Th(E)$ is an extension of (A, D) iff

$$Th(E) = \bigcup_{i=0}^{\infty} Th(E_i)$$

$$E_i \quad \alpha, \alpha = \{a_1, a_2, \ldots, a_n\}$$

$$\forall a_i \in \alpha : E_i \quad a_i$$

$$E \quad \neg\beta, \beta = \{b_1, b_2, \ldots, b_n\}$$

$$\forall b_i \in \beta : E \quad \neg b_i$$

- ❑ Non-constructive definition, since ***Th*(*E*)** is already used in each „iteration step“
  - ➪ however, each extension has the form

    ***Th*(*W* U *Con*(*D´*))**

    for a set of so-called **generating defaults *D´*, *D´*   *D***
  - ➪ simple „generate & test“ algorithm:
    - „generator“: compute powerset of ***Con*(*D*)** and „test“ each subset
    - „tester“: use definition to check if candidate is indeed an extension
  - ➪ more efficient algorithms see Baader & Hollunder
- ❑ α, β_i, γ are ABoxes

  $$\alpha \in Th(E_i) \Leftrightarrow E_i \quad \alpha$$

  $$\neg\beta \notin Th(E) \Leftrightarrow E \quad \neg\beta$$

ABox Axiom Entailment

A restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox axiom x is logically entailed by a restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox A,

$$A \models x, \quad \text{iff} \quad \begin{cases} x = a : C \longrightarrow \neg SAT(A \cup \{a : \neg C\} \\ x = (a,b) : \exists(u)(v).P \longrightarrow \\ \quad \neg SAT(A \cup \{(a,b) : \exists(u)(v).\overline{P}\}) \wedge \\ \quad \neg SAT(A \cup \{a : \forall u.\top\}) \wedge \\ \quad \neg SAT(A \cup \{b : \forall v.\top\}) \end{cases}$$

$SAT(A)$ decides the ABox consistency problem for an ABox A, and $u = v = has\_area$.

- ABox axiom entailment reduced to ABox consistency (negation necessary)
  - α, β_i, γ may only contain
    - concept membership axioms: „instance checking“ problem
    - complex role assertions (cannot be negated, but entailment can be decided)
    - other kinds of axioms possible?

Conclusion & Future Work

- Extension to Baader & Hollunder
  - ➪ ABox patterns
    - refer to specific individuals
    - complex role assertions
- Other kinds of ABox axioms?
  - ➪ however, concept membership assertions and complex role assertions sufficent in our application domain
- Default theories with specificity
  - ➪ if more than one default applicable, apply most specific first
  - ➪ additional partial ordering on defaults
  - ➪ S-Extensions instead of R-Extensions

$$d_1 \prec d_2 \Leftrightarrow \alpha(d_1) \quad \alpha(d_2) \wedge \alpha(d_2) \quad \alpha(d_1)$$

- Autoepistemic description logics (operators **A** and **K**)?
- Implementation
  - ➪ more efficient algorithms for computing extensions