On Terminological Default Reasoning about Spatial Information

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- Motivation
- Preliminaries
  - spatioterminological reasoning with $ALC\mathcal{R}P(S_2)$ [Lutz, Haarslev & Möller]
  - nonmonotonic reasoning with Reiter’s default logic
    - terminological default rules / theories [Baader & Hollunder]
- Spatioterminological default reasoning
  - on computing extensions
- Conclusion & future work
Motivation: Incomplete Spatioterminological Knowledge

- Combination of terminological, spatial & default reasoning techniques
  - Geographic Information Systems (GIS)
- Terminological knowledge
  - capital_city . city
- Spatial knowledge
  - properties of spatial relationships, eg. tpp (contains) is transitive
- Spatioterminological knowledge
  - a city is contained within exactly one country
  - two countries never overlap each other
- Default knowledge
  - data augmentation / completion
  - "b" could possibly be a city or a lake, but not both (disjoint concepts)
  - ABox realization would not work
Spatioterminological Reasoning with $\text{ALCRP}(S_2)$

- $\text{ALCRP}(D)$ extends $\text{ALC}(D)$
  - $\text{ALCRP}(D) = \text{ALC}(D) +$ role-forming predicate-based operator
  - decidable for restricted concept terms
  - restrictedness closed under negation

- $\text{ALCRP}(S_2)$
  - admissible concrete domain $S_2$, regular closed subsets of $\mathbb{R}^2$, called *regions*, with $\text{RCC8}$ predicates
  - properties of relationships captured by concrete domain, e.g. transitivity of $\text{tpp}$
  - $\text{RCC8}$ predicates
    - $\text{dc}$, $\text{ec}$, $\text{po}$, $\text{tpp}$, $\text{ntpp}$, $\text{tppi}$, $\text{ntppi}$, eq
  - defined roles, TBox axioms
    
    \begin{align*}
    \text{inside} & \doteq \exists (\text{has}_\text{area})(\text{has}_\text{area}).\text{tpp-ntpp} \\
    \text{contains} & \doteq \exists (\text{has}_\text{area})(\text{has}_\text{area}).\text{tppi-ntppi} \\
    \text{overlaps} & \doteq \exists (\text{has}_\text{area})(\text{has}_\text{area}).\text{po}
    \end{align*}

\[
(\exists(u_1, \ldots, u_n)(v_1, \ldots, v_m).P)^\mathcal{I} := \\
\{ (a, b) \in \Delta^\mathcal{I} \times \Delta^\mathcal{I} \mid \exists x_1, \ldots, x_n, y_1, \ldots, y_m \in \Delta^\mathcal{D} : \\
(a, x_1) \in u_1^\mathcal{I}, \ldots, (a, x_n) \in u_n^\mathcal{I}, \\
(b, y_1) \in v_1^\mathcal{I}, \ldots, (b, x_m) \in v_m^\mathcal{I}, \\
(x_1, \ldots, x_n, y_1, \ldots, y_m) \in P^\mathcal{D} \}\]
Spatioterminological Background Knowledge (TBox)

\[ \text{area} \triangleq \exists \text{has area}. \text{is-region} \]
\[ \text{country}_\text{region} \sqsubseteq \neg \text{natural}_\text{region} \sqcap \text{large}_\text{scale} \sqcap \text{area} \]
\[ \text{city}_\text{region} \sqsubseteq \neg \text{natural}_\text{region} \sqcap \neg \text{large}_\text{scale} \sqcap \text{area} \]
\[ \text{lake}_\text{region} \sqsubseteq \text{natural}_\text{region} \sqcap \text{area} \]
\[ \text{country} \triangleq \text{country}_\text{region} \sqcap \forall \text{contains} \neg \text{country}_\text{region} \sqcap \forall \text{overlaps} \neg \text{country}_\text{region} \sqcap \forall \text{inside} \neg \text{country}_\text{region} \]
\[ \text{city} \triangleq \text{city}_\text{region} \sqcap \exists \text{inside} \text{country}_\text{region} \]
\[ \text{lake} \sqsubseteq \text{lake}_\text{region} \]
Default Theories & Terminological Default Theories

- Default rules [Reiter, 1980]
  - $\alpha$ prerequisite, $\beta_i$ justifications, $\gamma$ conclusion, FOPL formulae
- Default theory $(W,D)$
  - $W =$ world description
  - $D =$ set of defaults
- Different sets of **extensions** of $(W,D)$
  - sceptical vs. credulous consequence
- Terminological default theories [Baader & Hollunder, 1991]
  - $\alpha, \beta_i, \gamma$ concept terms
  - $W =$ ABox, $D =$ set of closed default rules
  - restricted semantics, no skolemization
  - consequence problem decidable
- Closing concept terms over ABox $W$
  - concept terms $\Rightarrow$ ABox concept membership assertions

Example

$$\alpha : \beta_1, \beta_2, \ldots, \beta_n$$

$$\gamma$$

area : country

country

city

area : lake

lake
Closed Defaults

- Closing over $W$ yields 9 closed defaults
- Two extensions
  - $E1 = W \cup \{b : \text{city}, c : \text{lake}\}$
  - $E2 = W \cup \{b : \text{lake}, c : \text{lake}\}$
- $W \cup \{b : \text{lake}, c : \text{city}\}$ inconsistent, see picture
- 2 sets of „generating defaults“

World description $W =$
- $\{a : \text{country}, b : \text{area}, c : \text{area}, (a,b) : \text{contains}, (b,a) : \text{inside}, (a,c) : \text{overlaps}, (c,a) : \text{overlaps}\}$
We also want to conclude complex role assertions

\[ W = \{ \text{linköping: swedish\_city, sweden: country} \} \]
\[ E = W \cup \{ (\text{sweden, linköping}): \text{contains} \} \]

cannot be expressed with concept terms as \( \alpha, \beta_i, \gamma \)

"ABox patterns"

- ABoxes with variables, e.g. \( X, Y, Z \)
- to be closed over \( W \)
- can also refer to specific ABox individuals

"Duality"

- use known concept memberships to conclude spatial relationships
- use spatial relationships to deduce concept memberships

\[ \{ X : \text{swedish\_city}, \text{sweden: country} \} : \{ (\text{sweden, X}) : \text{contains} \} \]
\[ \{ (\text{sweden, X}) : \text{contains} \} \]
\[ \{ \text{linköping: swedish\_city, sweden: country} \} : \{ (\text{sweden, linköping}) : \text{contains} \} \]
\[ \{ (\text{sweden, linköping}) : \text{contains} \} \]
On Computing Extensions

Let $E$ be a set of closed formulae and $(A, D)$ be a closed default theory. We define $E_0 := A$ and for all $i \geq 0$

$$E_{i+1} := E_i \cup \{ \gamma \mid \alpha : \beta_1, \ldots, \beta_n / \gamma \in D, \alpha \in Th(E_i), \neg \beta_1, \ldots, \neg \beta_n \notin Th(E) \}. $$

Then, $Th(E)$ is an extension of $(A, D)$ iff

$$Th(E) = \bigcup_{i=0}^{\infty} Th(E_i)$$

$E_i \alpha, \alpha = \{a_1, a_2, \ldots, a_n\}$

$\forall a_i \in \alpha: E_i a_i$

$E \neg \beta, \beta = \{b_1, b_2, \ldots, b_n\}$

$\forall b_i \in \beta: E \neg b_i$

- Non-constructive definition, since $Th(E)$ is already used in each „iteration step“
  - however, each extension has the form

$$Th(W \cup Con(D'))$$

for a set of so-called generating defaults $D', D' \cup D$

- simple „generate & test“ algorithm:
  - „generator“: compute powerset of $Con(D)$ and „test“ each subset
  - „tester“: use definition to check if candidate is indeed an extension

- more efficient algorithms see Baader & Hollunder

- $\alpha, \beta, \gamma$ are ABoxes
  - $\alpha \in Th(E_i) \iff E_i \alpha$
  - $\neg \beta \notin Th(E) \iff E \neg \beta$
ABox Axiom Entailment

A restricted $\mathcal{ALC\overline{R\mathcal{P}}(S_2)}$ ABox axiom $x$ is logically entailed by a restricted $\mathcal{ALC\overline{R\mathcal{P}}(S_2)}$ ABox $A$, if

$$x = a : C \rightarrow \neg SAT(A \cup \{a : \neg C\})$$
$$x = (a, b) : \exists (u)(v). P \rightarrow$$
$$\neg SAT(A \cup \{(a, b) : \exists (u)(v). P\}) \land$$
$$\neg SAT(A \cup \{a : \forall u. T\}) \land$$
$$\neg SAT(A \cup \{b : \forall v. T\})$$

$SAT(A)$ decides the ABox consistency problem for an ABox $A$, and $u = v = has\_area$.

- ABox axiom entailment reduced to ABox consistency (negation necessary)
  - $\alpha, \beta_i, \gamma$ may only contain
    - concept membership axioms: „instance checking“ problem
    - complex role assertions (cannot be negated, but entailment can be decided)
    - other kinds of axioms possible?
Conclusion & Future Work

- Extension to Baader & Hollunder
  - ABox patterns
    - refer to specific individuals
    - complex role assertions
- Other kinds of ABox axioms?
  - however, concept membership assertions and complex role assertions sufficient in our application domain
- Default theories with specificity
  - if more than one default applicable, apply most specific first
  - additional partial ordering on defaults
  - S-Extensions instead of R-Extensions
    \[ d_1 \prec d_2 \iff \alpha(d_1) \land \alpha(d_2) \land \alpha(d_1) \]
- Autoepistemic description logics (operators $A$ and $K$)?
- Implementation
  - more efficient algorithms for computing extensions