Terminological Default Reasoning about Spatial Information: A First Step

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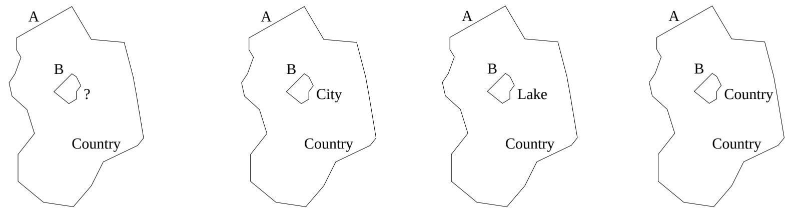
Abstract. We extend the theory about terminological default reasoning by using a logical base language that can represent spatioterminological phenomena. Based on this description logic language called $\mathcal{ALCRP}(\mathcal{S}_2)$, which is briefly introduced, we discuss an algorithm for computing so-called extensions ("possible worlds") of a world description and a set of defaults. We conclude with an application of the theory to problems in visual query systems and demonstrate the significance of the theory for spatioterminological reasoning in general and spatioterminological default reasoning in particular.

Keywords. Spatial Default Reasoning, Description Logics.

1 Introduction

For accessing spatial databases or geographic information systems (GIS), different query specification techniques have been proposed. For instance, the visual spatial query system VISCO developed in our group [9] can be used to query a spatial database (GIS) in a *visual* way. In contrast to conventional textual query systems the user is not required to learn a complicated textual query language in order to effectively use an information system. Users can query the database by drawing diagrammatic representations of what is to be retrieved from the spatial information system. However, experiences with the current VISCO system indicate that in the context of VISCO (and query systems in general), the specification of queries in a GIS still could be made easier by advances in research areas combining spatial and terminological reasoning.

First of all, the process of formulating (visual) queries can be facilitated by *automatically completing* queries in a meaningful way, therefore reducing the number of mouse interactions or –in the case of textual query languages– simplifying the composition of textual query elements. For instance, the process of selecting semantic concept descriptors for objects involved in a query (e.g. *city*, *lake*, *country*) can partly be automated by interpreting a partially specified query. In its current development stage, VISCO users can select concept descriptors from a list of over 300 predefined concepts. Thus, even a situation-adapted reduction of the complete list of possibilities to a suitable subset or an



(a) Incomplete Query (b) Possibility 1 (c) Possibility 2 (d) Inconsistency

Fig. 1. Automatic completion of visual queries by application of default rules.

order relation for sorting groups of possible concept candidates would be very appropriate.

The goal of this paper is to present solutions for representing domain knowledge concerning spatial as well as terminological reasoning for interpreting spatial structures (e.g. visual queries). Intuitively speaking, our solution to the specific query completion problems is to model so-called default knowledge that is used to make queries more precise if it can be applied in a consistent way. It is shown that there exists an algorithm for computing possible worlds (so-called extensions), i.e. the consequence problem for spatioterminological default logic is decidable.

In order to analyze the modeling problems in this context, we begin with a more detailed discussion of the visual query example. Let us assume the intention of a query is to retrieve *lakes* which are inside a particular *country* region. In Figure 1(a) the user just started to formulate the query. After he has specified that the type of the surrounding polygon A should be a *country*, the type of the small polygon B must be specified. As discussed above, a smart interface uses formal derivation processes for computing plausible candidates for "type specifications." For narrowing the set of possibilities we assume that two default rules are applicable: one saying that the interior small polygon B could be a *city* (Figure 1(b)) and another stating that B could be a *lake* (Figure 1(c)) if this does not lead to inconsistencies. Since an object can be either a *lake* or a *city* but not both, there is no way to believe in both possibilities at a time. This kind of default rule interaction is a simple example demonstrating the necessity of considering different *possible worlds* which must be maintained by the reasoning system. Depending on the default rule being used to conclude new knowledge, different subsequent conclusions might be possible.

Other potentially active default rules might be shown to produce inconsistencies with the set of current assertions without providing a possibility of using multiple worlds to avoid inconsistencies. For example, if there had been a default applied indicating that the small polygon B might as well be a *country*, we would have got a contradiction if we had an axiom (as part of our conceptual background knowledge) requiring that countries can never contain other countries. Thus, in our query context, the latter default cannot be applied and, as a consequence of computing and appropriately interpreting the set of possible worlds, we can compose a situation-adapted menu for the graphical user interface and

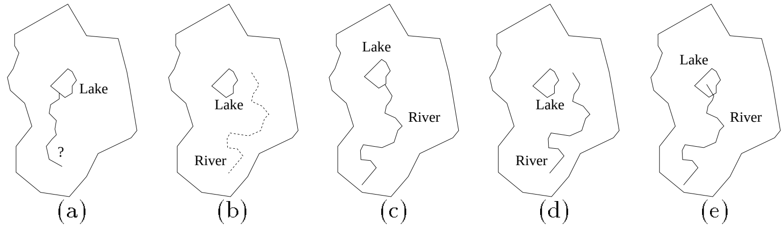


Fig. 2. Scenarios for situation-adapted completion of queries (see text).

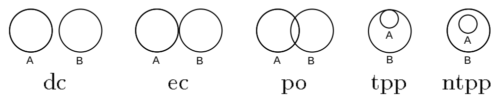


Fig. 3. Elementary relations between two regions A and B. The inverses of `tpp` and `ntpp` as well as the relation `eq` have been omitted.

the user can select between meaningful concepts for object B. In our specific example, the menu will contain items for *lake* and *city* but not for *country*.

If more than one possible world is computed, an intuitive criterion would be to select the world originating from a default with the more specific precondition or conclusion. E.g., in the query shown in Figure 2(a) we would prefer a default concluding that the thin graphical object might be a *river_flowing_into_a_lake* (which might be a useful concept in our scenario) instead of a “weaker” default concluding only that the object is an ordinary *river*.

The automatic augmentation of visual queries by conclusions of applied default rules can be seen as a specialization process. Therefore, this process might not only be useful during the construction of a visual query, but also useful as a tool for query refinement after a query has been executed that yields too many results. In addition, not only conceptual information is important. In a geographical information system context we also have to consider *spatial relations* between domain objects. An important example for spatial relations are topological relations. Due to its wide acceptance, we will rely on the well-known work about the RCC-8 relations modeling topological relations between non-empty regular closed subsets in R^n (see [4]). Figure 3 gives examples of the RCC-8 relations in the plane.

In the context of sketch-based visual querying, on the one hand it is sometimes useful to leave some spatial relations between graphical objects unspecified because they are unknown or simply because the user is not willing to specify them. On the other hand, in order to actually draw a picture, the user *must* specify each spatial relation, even if it is just one of several possible (base) relations. The problem of how to specify “don’t care relations” or “example relations” is well known and inherent in diagrammatic representations. It is similar to the problem of visualizing spatial disjunctions.

For example, in the query shown in Figure 2(b) we have a visual *disconnected* relation (`dc`) between the *river*[1] and the *lake*. If we intended the river to be

[1] Note that our river is assumed to be a thin *two-dimensional* object.

disjoint from the *lake*, the query answering system would not find any rivers flowing into this lake. The problem is how can we specify that the river should be within the country (non-tangential proper part, ntpp, or tangential proper part, tpp) but leave the relation to the lake unspecified. As a possible solution to this problem, we could simply *ignore* each visible dc relation. But, with this interpretation, we can now no longer state a query searching for rivers *not* flowing into this specific lake, which might be a very useful concept. We propose the following solution. For objects like the river that are drawn with a specific drawing attribute such as dashing, the universal spatial relation to other objects (disjunction of all base relations) is asserted. Dashed objects introduce no spatial query constraints. However, in some cases this would usually not match the users intention as there will be too many matches, i.e. the answer set will be too large. With the help of default knowledge we can automatically refine the query in a way that is appropriate according to the semantics of the objects involved in a query. So, we can guide the interpretation of spatial aspects by the help of conceptual background knowledge and application of defaults, yielding different hypotheses as possible worlds. A *river* flows into a *lake* or not, i.e. graphically both objects are either externally connected (relation ec, see also Figure 2(c)) or or they are disconnected (relation dc, see Figure 2(d)). With respect to a *lake*, there are no other possibilities. In our world model a *river* never overlaps with a *lake* (relation po, see also Figure 2(e)). This is assumed to be stated as an axiom as part of our general conceptual background knowledge. Besides defaults involving concept constraints we also have to take care of default rules with conclusions yielding new *relation* constraints.

The correct interpretation of the spatial relations explicit in a sketch depends on the conceptual background knowledge and demonstrates the benefits of integrated spatioterminological reasoning (see [6] for a first formal account on this topic). The important insight is the following duality: We can either use spatial relations between object pairs to conclude their concept memberships, or we can use already known concept memberships to conclude particular spatial relations between objects in the case of more general spatial relations (disjunctions of base relations). The conceptual background knowledge gives us the ability to conclude situation-specific candidates for spatial relations. In this paper, a formalization for this inference process is presented. Based on this description logic language called $\mathcal{ALCRP}(\mathcal{S}_2)$, which is briefly introduced, we discuss algorithms for computing so-called extensions ("possible worlds") of a world description and a set of defaults. We conclude with an application of spatioterminological default theory to problems in visual query systems and demonstrate the significance of the theory for spatioterminological reasoning in general and spatioterminological default reasoning in particular.

2 Modeling Conceptual and Spatial Information

We have seen the necessity for modeling conceptual background knowledge. The most widely accepted decidable formalisms with adequate expressiveness for this task are *description logics*. Basically, description logic formalisms distinguish between two kinds of building blocks: concepts and roles. Concepts denote sets of domain objects. Roles denote tuples of domain objects. As we have seen, in order to define meaningful concepts for spatial objects, it is also necessary to represent qualitative spatial relations and to exploit their various properties for reasoning. In particular, for a formalization of the motivating examples, we introduce a formalism for integrating reasoning about RCC-8 relations and reasoning about concepts. Since quantification over spatial relations is also needed for modeling (see below for examples), they should be represented as *roles* within a description logic formalism. In this section, we briefly introduce a description logic that supports this kind of modeling scheme. The logic is called $\mathcal{ALCRP}(\mathcal{S}_2)$ and is an instantiation of $\mathcal{ALCRP}(\mathcal{D})$ (see [6, 7] for an introduction). The name results from the well-known base language $\mathcal{ALC}(\mathcal{D})$ [1] and facilities for defining *R*oles based on *P*redicates.

2.1 Preliminaries

Based on the facilities offered by $\mathcal{ALCRP}(\mathcal{S}_2)$, roles representing RCC-8 relations can be defined and reasoned about using the formalism of "concrete domains" which provides an interface from a description logic reasoning system to another inference system possibly based on another theoretical background. The initial approach presented in [1] considered real numbers for engineering applications. The interface $\mathcal{D}$ is defined in terms of a pair of a domain $\Delta_\mathcal{D}$ and a set of names for predicates $P^\mathcal{D} \subseteq \Delta_\mathcal{D}^{arity(P)}$. For integrating the description logic part of $\mathcal{ALCRP}(\mathcal{D})$ (the abstract part) and the concrete part, the following admissibility criteria must hold. (1) The set of its predicate names must be closed under negation and must contain a name for a predicate *concrete_domain_top* for testing membership in $\Delta_\mathcal{D}$, (2) the satisfiability problem for finite conjunctions of predicates must be decidable. We briefly introduce the concrete domain $\mathcal{S}_2$ which can be used for representing two-dimensional spatial objects. Motivated by our introductory example we consider specific spatial objects whose spatial representations are given as polygons. $\mathcal{S}_2$ provides predicates that can be used to describe qualitative spatial RCC-8 relations as roles between spatial objects (see below for examples).

Definition 1. *The concrete domain* $\mathcal{S}_2$ *is defined w.r.t. the topological space* $\langle \mathbb{R}^2, 2^{\mathbb{R}^2} \rangle$. *The domain* $\Delta_{\mathcal{S}_2}$ *contains all non-empty, regular closed subsets of* $\mathbb{R}^2$ *which are called* regions *for short. The set of predicate names is defined as follows:*

- *A unary concrete_domain_top predicate* is-region *with* $\text{is-region}^{\mathcal{S}_2} = \Delta_{\mathcal{S}_2}$ *and its negation* is-no-region *with* $\text{is-no-region}^{\mathcal{S}_2} = \emptyset$.
- *The 8 basic predicates* dc, ec, po, tpp, ntpp, tppi, ntppi *and* eq *correspond to the RCC-8 relations. Due to space restrictions we would like to refer to [7] for a formal definition of the semantics.*

– *In order to name disjunctions of base relations, we need additional predicates. Unique names for these "disjunction predicates" are enforced by imposing the following canonical order on the basic predicate names:* dc, ec, po, tpp, ntpp, tppi, ntppi, eq. *For each sequence* $p_1, \ldots, p_n$ *of basic predicates in canonical order* ($n \geq 2$), *an additional predicate of arity 2 is defined. The predicate has the name* $\mathsf{p_1}\text{-}\cdots\text{-}\mathsf{p_n}$ *and we have* $(r_1, r_2) \in \mathsf{p_1}\text{-}\cdots\text{-}\mathsf{p_n}^{\mathcal{S}_2}$ *iff* $(r_1, r_2) \in \mathsf{p_1}^{\mathcal{S}_2}$ *or* $\ldots$ *or* $(r_1, r_2) \in \mathsf{p_n}^{\mathcal{S}_2}$. *The predicate* dc-ec-po-tpp-ntpp-tppi-ntppi-eq *is also called* spatially-related.
– *A binary predicate* inconsistent-relation *with* $\mathsf{inconsistent\text{-}relation}^{\mathcal{S}_2} = \emptyset$ *is the negation of* spatially-related.

Proposition 1. $\mathcal{S}_2$ *is admissible.*

Proof. This is proven in [7]. Based on the results presented in [8] we can conclude that there exists always a model whose individuals are polygons which are not necessarily internally connected.

In the following we define the syntax and semantics of role and concept terms in $\mathcal{ALCRP}(\mathcal{S}_2)$.

Definition 2. *Let* R *and* F *be disjoint sets of role and feature names, respectively. For brevity we also use the terms roles and features. Any element of* $\mathsf{R} \cup \mathsf{F}$ *is an* atomic *role term. A composition of features (written* $f_1 f_2 \cdots$*) is called a feature chain. A simple feature can be viewed as a feature chain of length 1. If* P *is a predicate name from* $\mathcal{S}_2$ *with arity* $n + m$ *and* $u_1, \ldots, u_n$ *as well as* $v_1, \ldots, v_m$ *are feature chains, then the expression* $\exists(u_1, \ldots, u_n)(v_1, \ldots, v_m).P$ (role-forming predicate restriction) *is a* complex *role term. Let* S *be a role name and let* T *be a role term. Then* $S \doteq T$ *is a* terminological axiom.

Definition 3. *Let* C *be a set of concept names which is disjoint to* R *and* F. *Any element of* C *is a* concept term (atomic *concept term). If* C *and* D *are concept terms,* R *is a role term,* P *is a predicate name from* $\mathcal{S}_2$ *with arity* n, *and* $u_1, \ldots, u_n$ *are feature chains, then the following expressions are also concept terms:* $C \sqcap D$ (conjunction), $C \sqcup D$ (disjunction), $\neg C$ (negation), $\exists R.C$ (exists restriction), $\forall R.C$ (value restriction), *and* $\exists u_1, \ldots, u_n.P$ (predicate exists restriction).

For all kinds of exists and value restrictions, the role term or the list of feature chains may be written in parentheses. Let A *be a concept name and let* D *be a concept term. Then* $A \doteq D$ *and* $A \sqsubseteq D$ *are terminological axioms as well. A finite set of terminological axioms* $\mathcal{T}$ *is a* terminology *or* TBox *if no concept or role name in* $\mathcal{T}$ *appears more than once on the left hand side of a definition and, furthermore, if no cyclic definitions are present.*

We now assign a meaning to $\mathcal{ALCRP}(\mathcal{S}_2)$ concept terms by giving a set-theoretic semantics as usual.

Definition 4. *An* interpretation $\mathcal{I} = (\Delta_{\mathcal{I}}, \cdot^{\mathcal{I}})$ *consists of a set* $\Delta_{\mathcal{I}}$ *(the abstract domain) and an interpretation function* $\cdot^{\mathcal{I}}$. *The sets* $\Delta_{\mathcal{S}_2}$ *and* $\Delta_{\mathcal{I}}$ *must be disjoint. The interpretation function maps each concept name* C *to a subset* $C^{\mathcal{I}}$ *of*

$\Delta_\mathcal{I}$, *each role name* R *to a subset* $R^\mathcal{I}$ *of* $\Delta_\mathcal{I} \times \Delta_\mathcal{I}$, *and each feature name* f *to a partial function* $f^\mathcal{I}$ *from* $\Delta_\mathcal{I}$ *to* $\Delta_{\mathcal{S}_2} \cup \Delta_\mathcal{I}$, *where* $f^\mathcal{I}(a) = x$ *will be written as* $(a, x) \in f^\mathcal{I}$. *If* $u = f_1 \cdots f_n$ *is a feature chain, then* $u^\mathcal{I}$ *denotes the composition* $f_1^\mathcal{I} \circ \cdots \circ f_n^\mathcal{I}$ *of the partial functions* $f_1^\mathcal{I}, \ldots, f_n^\mathcal{I}$. *Let the symbols* C, D, R, P, u_1, $\ldots$, u_m, *and* v_1, $\ldots$, v_m *be defined as in Definition 2 and 3, respectively. Then the interpretation function can be extended to arbitrary concept and role terms as follows:*

$$
\begin{aligned}
(C \sqcap D)^\mathcal{I} &:= C^\mathcal{I} \cap D^\mathcal{I}, (C \sqcup D)^\mathcal{I} := C^\mathcal{I} \cup D^\mathcal{I}, (\neg C)^\mathcal{I} := \Delta_\mathcal{I} \setminus C^\mathcal{I} \\
(\exists R.C)^\mathcal{I} &:= \{a \in \Delta_\mathcal{I} \mid \exists b \in \Delta_\mathcal{I} \colon (a, b) \in R^\mathcal{I}, b \in C^\mathcal{I}\} \\
(\forall R.C)^\mathcal{I} &:= \{a \in \Delta_\mathcal{I} \mid \forall b \in \Delta_\mathcal{I} \colon (a, b) \in R^\mathcal{I} \rightarrow b \in C^\mathcal{I}\} \\
(\exists u_1, \ldots, u_n.P)^\mathcal{I} &:= \{a \in \Delta_\mathcal{I} \mid \exists x_1, \ldots, x_n \in \Delta_{\mathcal{S}_2} \colon \\
&\qquad (a, x_1) \in u_1^\mathcal{I}, \ldots, (a, x_n) \in u_n^\mathcal{I}, (x_1, \ldots, x_n) \in P^{\mathcal{S}_2}\} \\
(\exists (u_1, \ldots, u_n)(v_1, \ldots, v_m).P)^\mathcal{I} &:= \{(a, b) \in \Delta_\mathcal{I} \times \Delta_\mathcal{I} \mid \\
&\qquad \exists x_1, \ldots, x_n, y_1, \ldots, y_m \in \Delta_{\mathcal{S}_2} \colon \\
&\qquad (a, x_1) \in u_1^\mathcal{I}, \ldots, (a, x_n) \in u_n^\mathcal{I}, \\
&\qquad (b, y_1) \in v_1^\mathcal{I}, \ldots, (b, y_m) \in v_m^\mathcal{I}, \\
&\qquad (x_1, \ldots, x_n, y_1, \ldots, y_m) \in P^{\mathcal{S}_2}\}
\end{aligned}
$$

An interpretation $\mathcal{I}$ *is a* model *of a TBox* $\mathcal{T}$ *iff it satisfies* $A^\mathcal{I} = D^\mathcal{I}$ $(A^\mathcal{I} \subseteq D^\mathcal{I})$ *for all terminological axioms* $A \doteq D$ $(A \sqsubseteq D)$ *in* $\mathcal{T}$. *A concept term* C *is* satisfiable *w.r.t. a TBox* $\mathcal{T}$ *iff there exists a model* $\mathcal{I}$ *of* $\mathcal{T}$ *such that* $C^\mathcal{I} \neq \emptyset$.

The following definition introduces the assertional language of $\mathcal{ALCRP}(\mathcal{S}_2)$, which can be used to represent knowledge about individual worlds.

Definition 5. *Let* O_{S_2} *and* O_A *be two disjoint sets of object names. If* C *is a concept term,* R *a role term,* f *a feature name,* P *a predicate name with arity* n, a *and* b *are elements of* O_A *and* x, *and* x_1, $\ldots$, x_n *are elements of* O_{S_2}, *then the following expressions are* assertional axioms.

$$a : C,\ (a, b) : R,\ (a, x) : f,\ (x_1, \ldots, x_n) : P$$

A finite set of assertional axioms is called ABox. *An* interpretation *for the concept language can be extended to the assertional language by additionally mapping every object name from* O_A *to a single element of* $\Delta_\mathcal{I}$ *and every object name from* O_{S_2} *to a single element from* $\Delta_{\mathcal{S}_2}$. *An interpretation satisfies an assertional axiom*

$$a : C \text{ iff } a^\mathcal{I} \in C^\mathcal{I}, \quad (a, b) : R \text{ iff } (a^\mathcal{I}, b^\mathcal{I}) \in R^\mathcal{I}, (a, x) : f \text{ iff } f^\mathcal{I}(a^\mathcal{I}) = x^\mathcal{I},$$
$$(x_1, \ldots, x_n) : P \text{ iff } (x_1^\mathcal{I}, \ldots, x_n^\mathcal{I}) \in P^\mathcal{D}$$

An interpretation is a model *of an ABox* $\mathcal{A}$ *w.r.t. a TBox* $\mathcal{T}$ *iff it is a model of* $\mathcal{T}$ *and furthermore satisfies all assertional axioms in* $\mathcal{A}$. *An ABox is* consistent *w.r.t. a TBox* $\mathcal{T}$ *iff it has a model.*

Satisfiability of concept terms can be reduced to ABox consistency as follows: A concept term C is satisfiable iff the ABox $\{a : C\}$ is consistent. Another basic problem is to decide whether an assertional axiom x is logically entailed by an ABox A, $A \models x$, i.e. all models of A are also models of x. If x is an assertional axiom $i : C$, this is called the *instance checking problem.* If ABox consistency is decidable, instance checking can be reduced to checking whether $A \cup \{i : \neg C\}$ is inconsistent.

In [6] it is shown that, unfortunately, the inference problem of checking the consistency of ABoxes in the "generic" language $\mathcal{ALCRP}(\mathcal{D})$ is undecidable in general. However, in [7] a restricted variant of $\mathcal{ALCRP}(\mathcal{D})$ is described that is indeed decidable if only (syntactically) *restricted* concept terms are used. Thus, the above-mentioned $\mathcal{ALCRP}(\mathcal{S}_2)$ inference problems can be decided if restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ concept terms are considered.

Definition 6. *A concept term X is called* restricted *w.r.t. a TBox $\mathcal{T}$ iff its equivalent X' which is unfolded w.r.t. $\mathcal{T}$ and in negation normal form fulfills the following conditions:*[2]

(1) For any subconcept term C of X' that is of the form $\forall R_1.D$ ($\exists R_1.D$) where R_1 is a complex role term, D does not contain any terms of the form $\exists R_2.E$ ($\forall R_2.E$) where R_2 is also a complex role term.

(2) For any subconcept term C of X' that is of the form $\forall R.D$ or $\exists R.D$ where R is a complex role term, D contains only predicate exists restrictions that (i) quantify over attribute chains of length 1 and (ii) are not contained inside any value and exists restrictions that are also contained in D.

A terminology is called restricted iff all concept terms appearing on the right-hand side of terminological axioms in $\mathcal{T}$ are restricted w.r.t. $\mathcal{T}$. An ABox $\mathcal{A}$ is called restricted w.r.t. a TBox $\mathcal{T}$ iff $\mathcal{T}$ is restricted and all concept terms used in $\mathcal{A}$ are restricted w.r.t. the terminology $\mathcal{T}$.

Proposition 2. *The ABox consistency problem for restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ concept terms is decidable.*

Proof. See Proposition 1 and [7].

Proposition 3. *The set of restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ concept terms is closed under negation.*

Proof. See [7].

These results will be needed for the default reasoning algorithms dealing with $\mathcal{ALCRP}(\mathcal{S}_2)$ concept and role terms (see below). The use of $\mathcal{ALCRP}(\mathcal{S}_2)$ for spatioterminological domain modeling is demonstrated in the following sections. The examples discussed here provide a formalization of the examples used in the introduction and will subsequently be exploited to illustrate spatioterminological reasoning with defaults.

[2] For technical reasons, we assume that a concept term is a subconcept term of itself. Any concept term can be transformed into an unfolded form by iteratively replacing concept and role names by their defining terms. An unfolded term is in negation normal form if negation is used only for concept names (for details see [7]).

2.2 Putting $\mathcal{ALCRP}(\mathcal{S}_2)$ to Work

Suppose we have the following $\mathcal{ALCRP}(\mathcal{S}_2)$ TBox supplying our conceptual background knowledge. First, we define roles according to the spatial relations needed in our application example. As an ontological decision we agree upon using the feature *has_area* for referring to the spatial representation of individuals.

$$\begin{aligned} inside &\doteq \exists(has\_area)(has\_area).\mathsf{tpp\text{-}ntpp} \\ contains &\doteq \exists(has\_area)(has\_area).\mathsf{tppi\text{-}ntppi} \\ overlaps &\doteq \exists(has\_area)(has\_area).\mathsf{po} \\ touches &\doteq \exists(has\_area)(has\_area).\mathsf{ec} \\ disjoint &\doteq \exists(has\_area)(has\_area).\mathsf{dc} \end{aligned}$$

In addition, we give the definition of concepts required to model domain objects representing different kinds of regions in a TBox that satisfies the $\mathcal{ALCRP}(\mathcal{S}_2)$ restrictedness criteria.

$$\begin{aligned} area &\doteq \exists has\_area.\mathsf{is\text{-}region} \\ natural\_region &\doteq \neg administrative\_region \\ country\_region &\stackrel{.}{\sqsubseteq} administrative\_region \sqcap large\_scale \sqcap area \\ city\_region &\stackrel{.}{\sqsubseteq} administrative\_region \sqcap \neg large\_scale \sqcap area \\ lake\_region &\stackrel{.}{\sqsubseteq} natural\_region \sqcap area \\ river\_region &\stackrel{.}{\sqsubseteq} natural\_region \sqcap area \end{aligned}$$

An *area* is a two-dimensional region with some extent. Furthermore, we distinguish between *administrative_regions* and *natural_regions* which are disjoint concepts. The difference between a *country_region* and a *city_region* is that the former is *large_scale*, but the latter is not. Thus, these two concepts are disjoint as well. The intention behind the other concepts should be obvious. We would like to mention that these region concepts are *basic concepts* being used to define a set of concepts which are used by a query interface system (e.g. VISCO). For demonstration purposes we consider some of the concepts that might be used in a full-fledged (visual) query system.

$$\begin{aligned} country &\doteq country\_region \sqcap \forall contains.\neg country\_region \sqcap \\ &\qquad \forall overlaps.\neg country\_region \sqcap \forall inside.\neg country\_region \\ city &\doteq city\_region \sqcap \exists inside.country\_region \\ lake &\stackrel{.}{\sqsubseteq} lake\_region \\ river &\doteq river\_region \sqcap \forall overlaps.\neg lake\_region \sqcap \forall inside.\neg lake\_region \end{aligned}$$

A *country* is a *country_region* and can never contain other *country_regions*. Also, *countries* never overlap other *country_regions*. Each *city* must belong to a specific *country*, i.e. must lie within a *country*. Unfortunately, we cannot write this directly as $\exists inside.country$ because the unfolded resulting term is no longer restricted. So, we have to use the somewhat weaker version with the base concept *country_region*. In our world model a *city* must be inside a *country*. For a *river* we require that it never *overlaps* or is *inside* with a *lake_region*.

$$river\_flowing\_into\_a\_lake \doteq river \sqcap \exists touches.lake\_region$$

A $river\_flowing\_into\_a\_lake$ is a specific $river$ that $touches$ a $lake\_region$ (please recall that the RCC-8 relations ec and po and also ec and ntpp-tpp are disjoint). It would be reasonable to also state that cities do not overlap other cities etc., but this is ignored here for the sake of brevity.

We have seen that $\mathcal{ALCRP}(\mathcal{S}_2)$ provides the necessary expressiveness to model domain objects in our geographic information system scenario. In the next section, these concepts will be augmented with defaults in order to demonstrate how the problems sketched in the introduction can be solved. In [7] more examples are given which also demonstrate the influence of spatial reasoning with RCC-8 relations on TBox reasoning (e.g. subsumption between concepts).

3 Spatioterminological Reasoning with Defaults

Let us now briefly review the theory about defaults and then show how to compute the different extensions of a closed terminological default theory in order to formalize the terms we already used informally.

3.1 Preliminaries

A default rule ("default" for short) has the form

$$\frac{\alpha : \beta_1, \beta_2, \ldots, \beta_n}{\gamma}$$

where α, β_i and γ are usually first-order formulae. Informally speaking, the idea behind these default rule is the following. α is called the *precondition* of the rule, the β_i terms are called *justifications,* and γ is the *consequent.* The formula γ is added to the world description when α is entailed by the world description and each formula β_i is consistent with the world description. In our case, α, β and γ are not arbitrary first-oder formulae, but $\mathcal{ALCRP}(\mathcal{S}_2)$ *concept terms* that fulfill the $\mathcal{ALCRP}(\mathcal{S}_2)$ restrictedness criteria. Because concept terms correspond to unary predicates ranging over a free variable, say x, these defaults are called *open defaults.* In contrast, *closed defaults* do not contain any free variables.

Using description logic terms in default rules instead of first-order or propositional logic formulae has first been considered in [2]. A *terminological default theory* is a pair (A, D) where A is an ABox, and D is a finite set of *terminological* default rules that have to be closed over the ABox A. These closed default rules can be obtained by instantiating the free variable x in the concept expressions with all explicitly mentioned ABox individuals. Default rules are never applied to implicit individuals that might be introduced by exists restrictions. Due to this semantics, skolemization as originally proposed by Reiter to treat open defaults is not necessary (see [2] for a discussion of problems with skolemization).

3.2 Solving the example problems

Recalling our introductory example, let us define the following *default rules*:

$$d_1 = \frac{area\ :\ city}{city}, \quad d_2 = \frac{area\ :\ lake}{lake}, \quad d_3 = \frac{area\ :\ country}{country}$$

Suppose we have an ABox $\{a : country, b : area, (a, b) : contains, (b, a) : inside\}$ corresponding to the visual query shown in Figure 1(a). Intuitively, answering the query means finding "equality assertions" that unify individuals in the query and in an ABox representing, for instance, a GIS database. Actually, the problem how to use an ABox as a query will be addressed in future work. Note that the unique name assumption does not hold for $\mathcal{ALCRP}(\mathcal{S}_2)$.

Closing defaults, i.e. instantiating the defaults d_1, d_2, d_3 over the ABox individuals a and b yields 6 different closed defaults. Now, let us assume α, β and γ have been replaced by the corresponding assertional axioms. We use the notation $d_i(ind)$ to refer to a default that is instantiated with the individual *ind*. Given our 6 closed default rules let us examine the status of each:

- Default $d_1(a)$ cannot be applied because adding $a : city$ to the ABox yields a contradiction with $a : country$. The concepts *country_region* and *city_region* are disjoint (due to *large_scale* and $\neg$*large_scale*).
- Default $d_1(b)$ can be applied. We get an augmented ABox or *extension one* corresponding to Figure 1(b):

$$\{a : country, b : area, b : city, (a, b) : contains, (b, a) : inside\}$$

- Default $d_2(a)$ cannot be applied because adding $a : lake$ to the ABox yields a contradiction with $a : country$. A *country* is an *administrative_region* and a *lake* is defined as a *natural_region*, and both are disjoint concepts.
- Default $d_2(b)$ can be applied. Thus, we can get an augmented ABox or *extension two*, corresponding to Figure 1(c):

$$\{a : country, b : area, b : lake, (a, b) : contains, (b, a) : inside\}$$

  However, if we have an ABox already augmented by the conclusion of default $d_1(b)$, $b : city$, we cannot apply $d_2(b)$. So, only one of $d_1(b)$ or $d_2(b)$ can be applied, resulting in two different *extensions.*
- Default $d_3(a)$ cannot be applied, because its conclusion is already entailed by the ABox.
- Default $d_3(b)$ cannot be applied even if no other default has been applied before. Adding the default's consequent $b : country$ would yield an inconsistent ABox because a is already known to be a *country* and so, among others, $a : \forall contains.\neg country\_region$ holds. Because $(a, b) : contains$ holds and $b : country$ would imply $b : country\_region$, the default cannot be applied. Thus, we cannot get an extension corresponding to the wrong interpretation in Figure 1(d).

Another subtle inference can be demonstrated by showing that the default $d_2(b)$ cannot be applied to conclude that object b in Figure 4 is a *city*. Trying to do so would result in a constraint $b : city \sqcap \exists inside.country\_region$. Therefore, polygon a cannot be the appropriate *country_region* because $(b, a) : overlaps$ holds. Due to the exists restriction there exists an implicit individual c which is a *country_region* such that $(b, c) : inside$ holds. As can be seen in Figure 4,

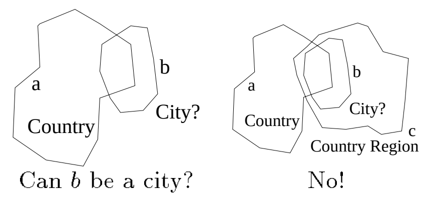


Fig. 4. Subtle inferences due to topological constraints.

there is no way to find a spatial arrangement such that b is inside c and c does not overlap with a or does not contain a. Because a is a *country* and, therefore, may not overlap or may not be contained in another *country_region*, there is no way to conclude that b could possibly be a *city*.

3.3 Reasoning with Spatioterminological Default Theories

Intuitively, given a closed terminological default theory (A, D) a deductively closed set of consequences of such a theory is referred to as an *extension*. As usual, the exact definition is given by a fixpoint construction. We cite a formal definition taken from [2]. $Th(\Gamma)$ stands for the deductive closure of a set of formulae Γ. In a description logic context Γ is an ABox.

Definition 7. *Let E be a set of closed formulae and (A, D) be a closed default theory. We define $E_0 := A$ and for all $i \geq 0$*
$E_{i+1} := E_i \cup \{\gamma \mid \alpha : \beta_1, \ldots, \beta_n/\gamma \in D, \alpha \in Th(E_i),\ \neg\beta_1, \ldots, \neg\beta_n \notin Th(E)\}$.
Then, $Th(E)$ is an extension of (A, D) iff $Th(E) = \bigcup_{i=0}^{\infty} Th(E_i)$.

Depending on the reasoning mode the *consequence problem* for terminological default theories is to decide whether a given assertional axiom is member of all extensions (skeptical mode) or of at least one extension (credulous mode).

In order to be able to infer spatial relations between domain objects, the basic terminological default reasoning approach described in [2] is adapted. The basic idea is that the precondition, the justifications and the consequent of a default can be ABoxes with complex role axioms.

Definition 8. *A* spatioterminological default rule *d (or spatioterminological default for short) has the form $d = \alpha : \beta_1 \ldots \beta_n/\gamma$ where α, β_i and γ are consistent ABoxes whose unfolded versions contain only concept axioms with restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ concept terms and only predicate-based role axioms of the form $(x, y) : \exists(has\_area)(has\_area).P$ with P being an $\mathcal{S}_2$ predicate of arity two. A* spatioterminological default theory *is a tuple (A, D) where D is a set of spatioterminological default rules and A is a consistent and restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox.*

Lemma 1. *A restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox axiom x is logically entailed by a restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox A,*

$$A \models x, \quad \textit{iff} \quad \begin{cases} x = a : C \longrightarrow \neg SAT(A \cup \{a : \neg C\}) \\ x = (a,b) : \exists(u)(v).P \longrightarrow \\ \quad \neg SAT(A \cup \{(a,b) : \exists(u)(v).\overline{P}\}) \wedge \\ \quad \neg SAT(A \cup \{a : \forall u.\top\}) \wedge \\ \quad \neg SAT(A \cup \{b : \forall v.\top\}) \end{cases}$$

$SAT(A)$ decides the ABox consistency problem for an ABox A, and $u = v = has\_area$.

Proof. The first case is the instance checking problem, which is decidable because C is a restricted concept term. The second case is more problematic, because the $\mathcal{ALCRP}(\mathcal{S}_2)$ language does not provide a negation operator for predicate-based role axioms. However, we can check whether $(a,b) : \exists(has\_area)(has\_area).\overline{P} \vee a : \neg\exists has\_area.\textsf{is-region} \vee b : \neg\exists has\_area.\textsf{is-region}$ holds. The NNF of $\neg\exists has\_area.\textsf{is-region}$ is $\exists has\_area.\textsf{is-no-region} \sqcup \forall has\_area.\top$. Since $\exists has\_area.\textsf{is-no-region}$ is inconsistent, the resulting term is $(a,b) : \exists(has\_area)(has\_area).\overline{P} \vee a : \forall has\_area.\top \vee b : \forall has\_area.\top$. Obviously, this is not an $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox. However, $A \cup \{a_1 \vee a_2 \vee \cdots \vee a_n\}$ is inconsistent iff $\forall a_i : A \cup \{a_i\}$ is inconsistent. Note that the predicate name $\overline{P}$ exists because the concrete domain is required to be admissible.

Theorem 1. *The consequence problem for a spatioterminological default theory (A, D) is decidable.*

Proof. Considering the sound and complete tableaux calculus for deciding the consistency of restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes, $x \in Th(\Gamma)$ iff $\Gamma \models x$. Thus, instead of taking $Th(E)$ we can view the ABox E as a representative for an extension. The fixpoint construction in Definition 7 can be used as a tester for determining whether a given ABox E really *is* an extension of a default theory (A, D). Since each extension E is an ABox having the form $A \cup \{\gamma \mid \alpha : \beta_1 \ldots \beta_n / \gamma \in D'\}$ for a set of so-called *generating defaults* $D' \subseteq D$, we can simply check for each element E of $\{A \cup X \mid X \in 2^{\{\gamma \mid \alpha:\beta_1 \ldots \beta_n/\gamma \in D\}}\}$ whether it is an extension or not. The following inference problems need to be decided:

1. $\alpha \in Th(E_i)$: This can be easily tested by checking whether $E_i \models \alpha$ where $\alpha = \{a_1, a_2, \ldots, a_n\}$. We can decide this *ABox entailment problem* iff we can decide whether each assertional axiom a_i follows from A, i.e. $\forall a_i \in E_i : A \models a_i$. This can be decided according to Lemma 1 because the elements of α are restricted to be concept axioms or predicate-based role axioms.
2. $\neg\beta_i \notin Th(E_i)$: This can be checked by testing whether $E \not\models \neg\beta_i$. More generally, $A \not\models \neg B$, where $B = \{b_1, b_2, \ldots, b_n\}$ iff $\forall b_i \in B : A \not\models \neg b_i$. However, $A \not\models \neg b_i$ iff $A \cup \{b_i\}$ is consistent. The ABox consistency problem for restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes is decidable.

3. $Th(E) = \bigcup_{i=0}^{\infty} Th(E_i)$: The fixpoint can be constructed in a finite number of steps because we consider only a finite number of defaults. In principle, we have to decide the *ABox equivalence problem*. An ABox A_1 is equivalent to an ABox A_2, $A_1 \equiv A_2$ iff $A_1 \models A_2$ and $A_2 \models A_1$, i.e. the ABox equivalence problem can be reduced to two ABox entailment problems. Unfortunately, considering $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes there might not only be concept axioms and predicate-based role axioms in A_1 or A_2 but also role axioms of the form $x = (a, b) : R$ or $x = (a, b) : f$ or $x = (x_1, \ldots, x_n) : P$ where R is a role name, f is a feature and P is an $\mathcal{S}_2$ predicate of arity n. In this case Lemma 1 is not applicable. However, both $A_1 (= E)$ and $A_2 (= E_n)$ are constructed on the basis of A, that is, we have to decide whether two ABoxes of the form $A_1 = A \cup \Gamma_1$ and $A_2 = A \cup \Gamma_2$ are equivalent, where $\Gamma_i \subseteq \{\gamma \mid \alpha : \beta_1 \ldots \beta_n / \gamma \in D\}$. Obviously, $(A \cup \Gamma_1) \equiv (A \cup \Gamma_2)$ iff $A \cup \Gamma_1 \models \Gamma_2$ and $A \cup \Gamma_2 \models \Gamma_1$. Since both Γ_1 and Γ_2 contain only concept axioms and predicate-based role axioms, Lemma 1 is applicable.

In [2] another algorithm is discussed for computing extensions. This algorithm seems to be more efficient in the average case. There is a strong conjecture that the algorithm is also applicable in the $\mathcal{ALCRP}(\mathcal{S}_2)$ context. Furthermore, it can easily be seen that the results for spatioterminological default theories wrt. $\mathcal{ALCRP}(\mathcal{S}_2)$ can be extended to $\mathcal{ALCRP}(\mathcal{D})$ as well.

3.4 Applying Spatioterminological Default Theories

We have already used spatial relations to conclude possible concept memberships per default. A major achievement of our theory is that it is possible to conclude spatial relations between objects. Recalling our introductory example, we would like to be able to conclude that possible spatial relations between the river and the lake in Figure 2(b) are either ec (touches) or dc (disconnected). These conclusions cannot be expressed with the limited terminological default rules introduced in [2] because there α, β and γ are *concept expressions*. Although in [2] it is possible to conclude exists restrictions, this cannot be used to infer spatial relations between *specific* individuals. For the river in our example, we could conclude $\exists touches.lake$. However, this does not require that the existing lake must coincide with the lake which we specified in our graphical query. We therefore extended the terminological default rules by substituting the concept expressions α, β and γ by so-called ABox patterns. These ABox patterns are basically ABoxes with placeholders (variables such as x, y etc.) for individuals. Closing the default rules instantiates the patterns with all possible combinations of individuals yielding ordinary $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes. We can also refer to *specific* individuals (for instance, an individual lake such as "Bodensee").

Returning to our example, we could, in principle, define a single default to conclude $(lake, river) : \exists(has\_area)(has\_area).\mathsf{dc\text{-}ec}$, but if we want to reflect the default's conclusion at the user interface level, we must use two different defaults, concluding different RCC-8 base relations, corresponding to two different completions of the visual query:

$$d_4 = \frac{\{x : lake, y : river, (x, y) : spatially\_related\} \;:\; \{(x, y) : disjoint\}}{\{(x, y) : disjoint\}},$$

$$d_5 = \frac{\{x : lake, y : river, (x, y) : spatially\_related\} \;:\; \{(x, y) : touches\}}{\{(x, y) : touches\}}$$

Closing the patterns, i.e. instantiating x, y over the ABox $A = \{l : lake, r : river\}$ would yield 8 different closed defaults. Since ec and dc are disjoint RCC-8 relations, only d_4 or d_5 can be applied. Note that, if we had $A = \{l : lake, r : \neg river\_flowing\_into\_a\_lake, r : river\}$, default d_5 could not be applied.

3.5 A Note on Terminological Default Reasoning with Specificity

If it were already known that *river* is really a *river_flowing_into_a_lake* and we already specified a lake in our graphical query, we would like to conclude that the lake in the query should be *the* lake. If we specified more than one lake in our graphical query but only one river, different possibilities could be visualized and searched for. Note that this interpretation of the graphical query would not be possible without don't care relations.

$$d_6 = \frac{\{x : lake, y : river\_flowing\_into\_a\_lake\} \;:\; \{(x, y) : touches\}}{\{(x, y) : touches\}}$$

Since $(x, y) : spatially\_related$ is already implied for x and y, we omitted this constraint. In the case of d_6, we would like to render the application of d_4 and d_5 *invalid*, because they are "less specific" than d_6 (even if d_5 yields the same conclusion, *touches*).

A default d_a is said to be more specific than d_b, $d_a \prec d_b$ iff $(\alpha(d_a) \models \alpha(d_b)) \wedge (\alpha(d_b) \not\models \alpha(d_a))$ where $\alpha(D)$ denotes the precondition of the default D. Algorithms for computing the so-called *S-extensions* (S for specificity) have already been developed by Baader and Hollunder [3]. There is a strong conjecture that these algorithms can be applied in our $\mathcal{ALCRP}(\mathcal{S}_2)$ context as well. In contrast, the ordinary extensions are called *R-extensions* (R for Reiter). In our example, we would get two different R-extensions, but only one S-extension containing the ABox axiom $(r, l) : touches$. The other *R-extension* containing $(r, l) : disjoint$ could not be derived, since only the most specific active defaults are applied when computing extensions. This would render the application of d_4 and d_5 impossible because d_6 is also active and more specific than both d_4 and d_5.

4 Conclusion

To the best or our knowledge we have proposed a first theory for spatio-terminological default reasoning. Our new spatio-terminological default theory extends previous work done in [2] and [3]. The new contributions are: As a base language, the expressive spatio-terminological description logic $\mathcal{ALCRP}(\mathcal{S}_2)$ is used. Allowing not only concept terms as formulae occurring inside default rules but also $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes with assertional predicate-based role axioms is necessary from an application-oriented point of view but imposes a number of theoretical problems. We have shown that extensions of a closed $\mathcal{ALCRP}(\mathcal{S}_2)$ spatio-terminological default theory can be effectively computed. Although the basic algorithm discussed in this paper might not be directly used in applications, there is a strong conjecture that the techniques used in the basic algorithm presented in this paper can also be used in the more efficient algorithms proposed in [2]. We have demonstrated that interesting application problems concerning spatio-terminological default knowledge can be solved with the new theory.

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