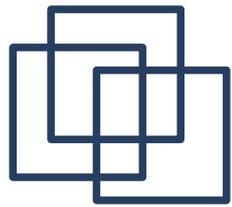


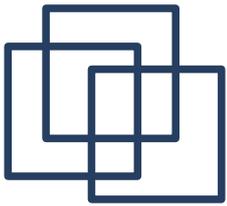
Introduction to DLs, OWL, RacerPro





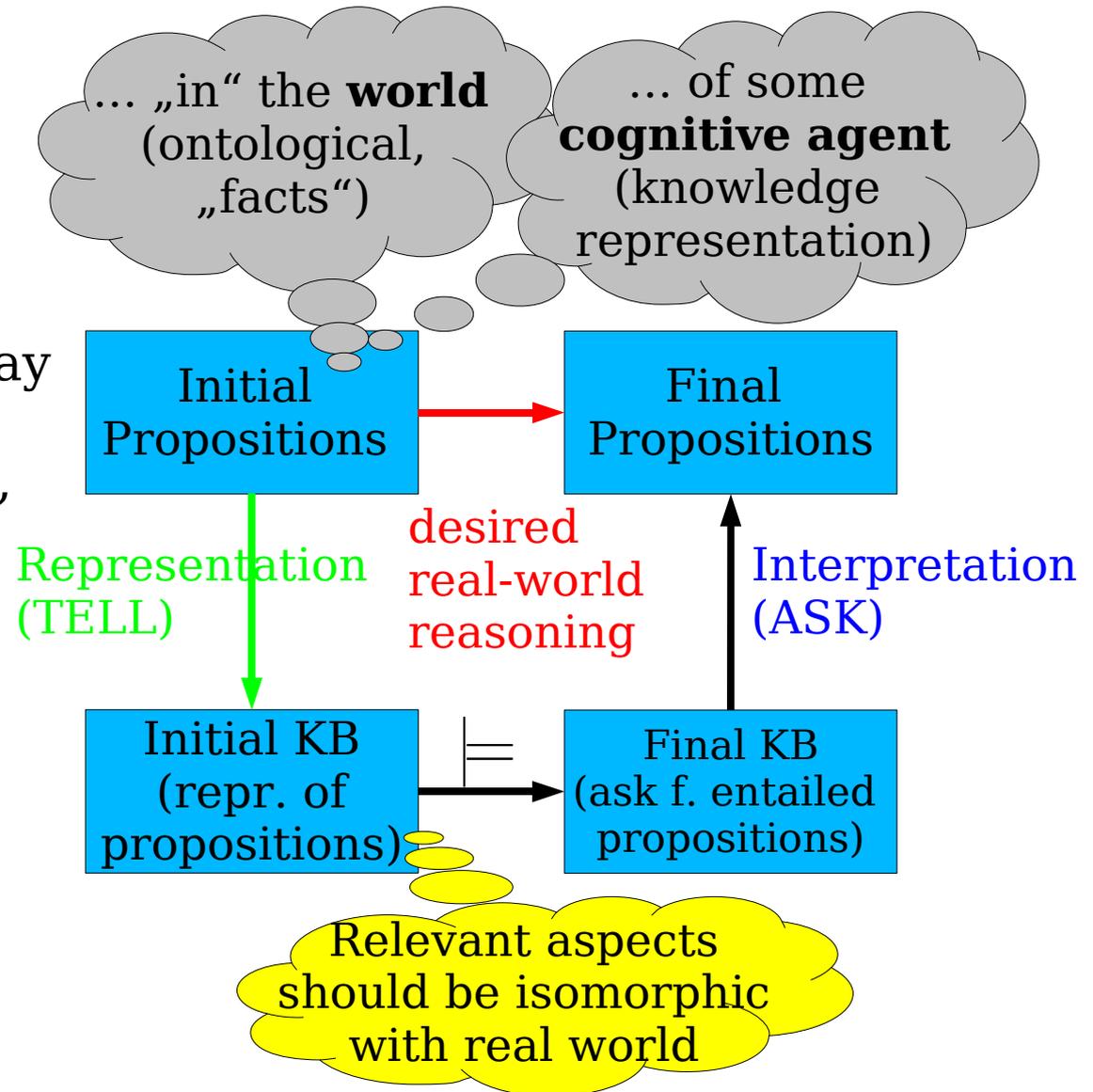
Logic-Based Knowledge Representation

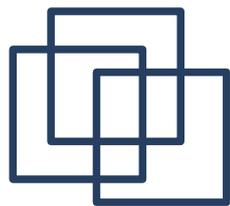
- DLs: family of **logics** for **knowledge representation (KR)**
 - foundation for ontologies & Semantic Web
 - ... but what is logic-based KR?
- Logic
 - formal syntax and semantics
 - notion of entailed / logically implied formulas: \models
 - mechanical reasoning (inference / proof system)
- Basic idea of logic-based KR
 - knowledge base (KB) = set of formulas (axioms)
 - represents knowledge of some „agent“
 - agent uses proof system to derive conclusions from the KB which are meaningful in the environment



Purpose of (Logical) Models

- Replace „real-world reasoning“ in some DOD with computational operations performed on the representations (\models)
- „real-world“ reasoning may be impossible, too dangerous, too expensive, too complicated, ...
- Representation involves **abstraction**
 - conceptualization!
- Models have a **purpose**
 - conceptualization depends on purpose and DOD

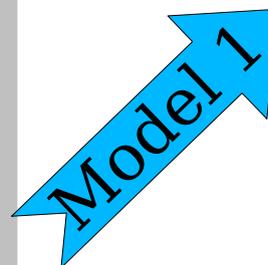




KBs, Logical Models, Entailment

KB = Set of Formulas / Axioms

- All individuals are female or male
- Mothers are parents and woman
- A parent has a child
- Woman are female persons
- Betty is a mother

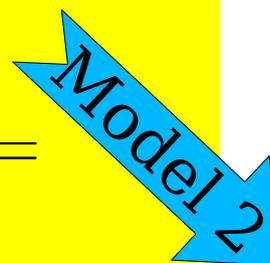


female
person
mother
parent *female*



Relational Structure 1

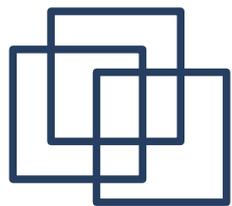
- Implicit information, things can be left unsaid (e.g., that betty is a person)
- What holds in all models? Entailment =
- The more axioms, the less models
- The less models, the more entailed formulas, the more implicit / entailed information!
- Chaos = absence of structure



female
person
mother
parent *male*



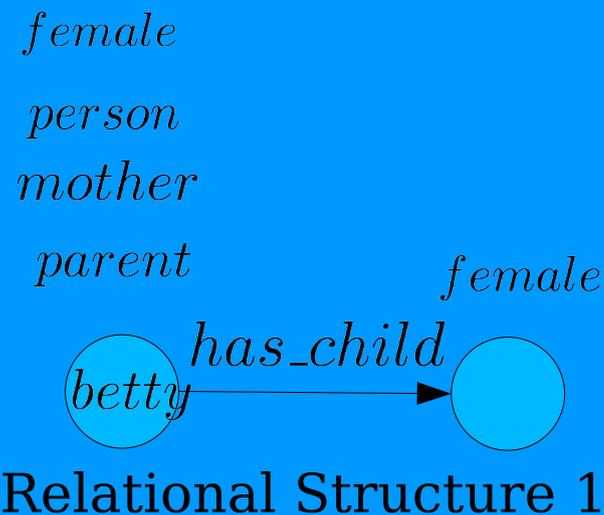
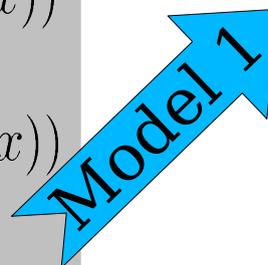
Relational Structure 2



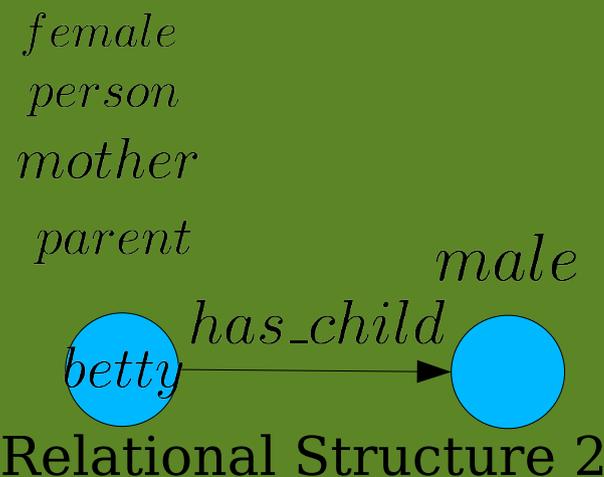
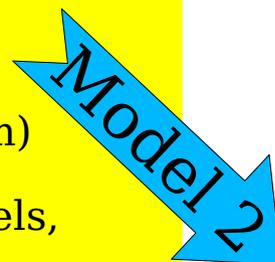
KBs, Logical Models, Entailment (2)

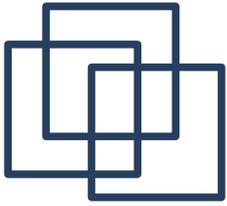
KB = Set of Formulas / Axioms

$\forall x.(female(x) \vee male(x))$
 $\forall x.(mother(x) \rightarrow parent(x) \wedge woman(x))$
 $\forall x.(parent(x) \leftrightarrow \exists y.(has_child(x, y)))$
 $\forall x.(woman(x) \leftrightarrow female(x) \wedge person(x))$
 $mother(betty)$



- Some KBs have only infinite models; countable infinite models suffice
- For each first-order logic model, there is a bigger one (Löwenheim-Skolem)
- Each KB has an infinite number of models, or is contradictory
- A contradictory KB has no models (important inference problem!)
- From a contradictory KB, everything follows





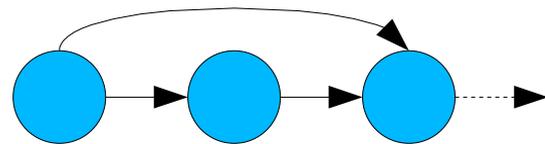
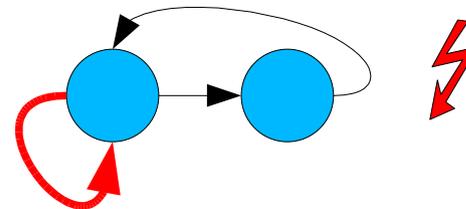
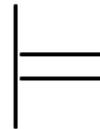
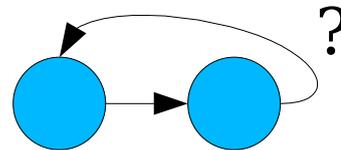
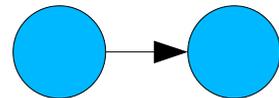
Infinite Models

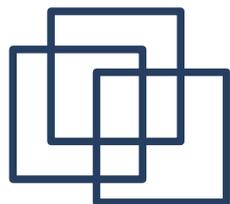
$$\forall x, y, z. has_ancestor(x, y) \wedge has_ancestor(y, z) \rightarrow has_ancestor(x, z)$$

$$\forall x. \neg has_ancestor(x, x)$$

$$\forall x. \exists y. has_ancestor(x, y)$$

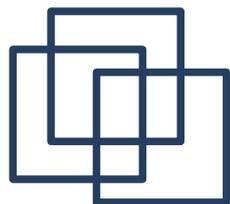
betty





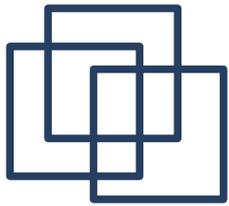
Why Description Logics?

- Formal
 - suitable as **ontology** languages (Gruber definition)
 - foundation for the Semantic Web
- Well-understood
 - Semantics, complexity, implementation techniques
- Decidable
 - unlike FOPL
- Relatively mature set of tools available
 - Reasoners: Fact++, Pellet, RacerPro
 - Editors: Protege, Swoop, RacerPorter, ...
 - Visualizers: OWLViz, OntoTrack, ...

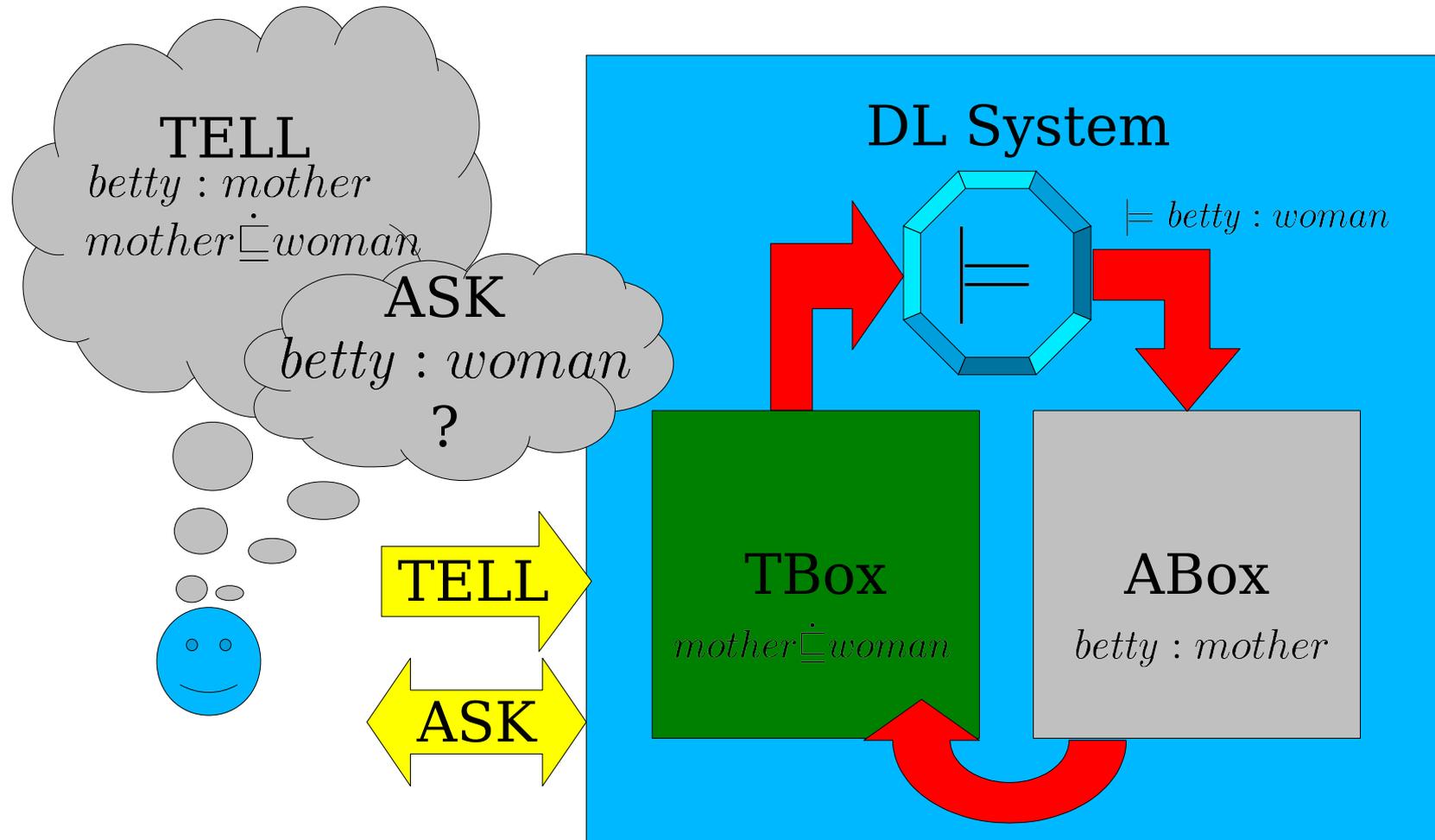


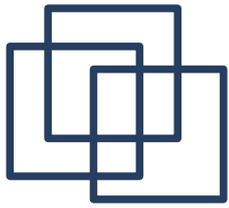
Description Logics : Basic Notions

- Based on first order-logic
 - but variable-free and decidable
 - concept languages, class-based KR
- Central notions:
 - Concept (OWL: Class)
 - atomic or complex (concept term)
 - Role (OWL: Property, RDF: Predicate)
 - Individual
 - Container data structures:
 - TBox: Set of terminological axioms
 - ABox: Set of assertional axioms



Architecture of a DL System





Description Logics : Concepts

- Represent „classes“ = sets of individuals
 - atomic concepts : basic vocabulary, e.g. *person*
 - complex concepts : e.g. $person \sqcap female$
- Semantics via interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$
 - interpretation of a concept = set of individuals in $\Delta^{\mathcal{I}}$
 - function $\cdot^{\mathcal{I}}$ maps concept C to subset of $\Delta^{\mathcal{I}}$
$$person^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \quad \cdot^{\mathcal{I}} : \mathcal{N}_C \mapsto 2^{\Delta^{\mathcal{I}}}$$
 - Top and bottom: $\top^{\mathcal{I}} = \Delta^{\mathcal{I}} \quad \perp^{\mathcal{I}} = \emptyset$
- Concept constructors, e.g. conjunction
 - constraint on interpretation of complex concepts
$$(person \sqcap female)^{\mathcal{I}} = person^{\mathcal{I}} \cap female^{\mathcal{I}}$$

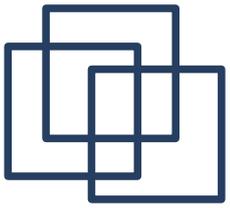
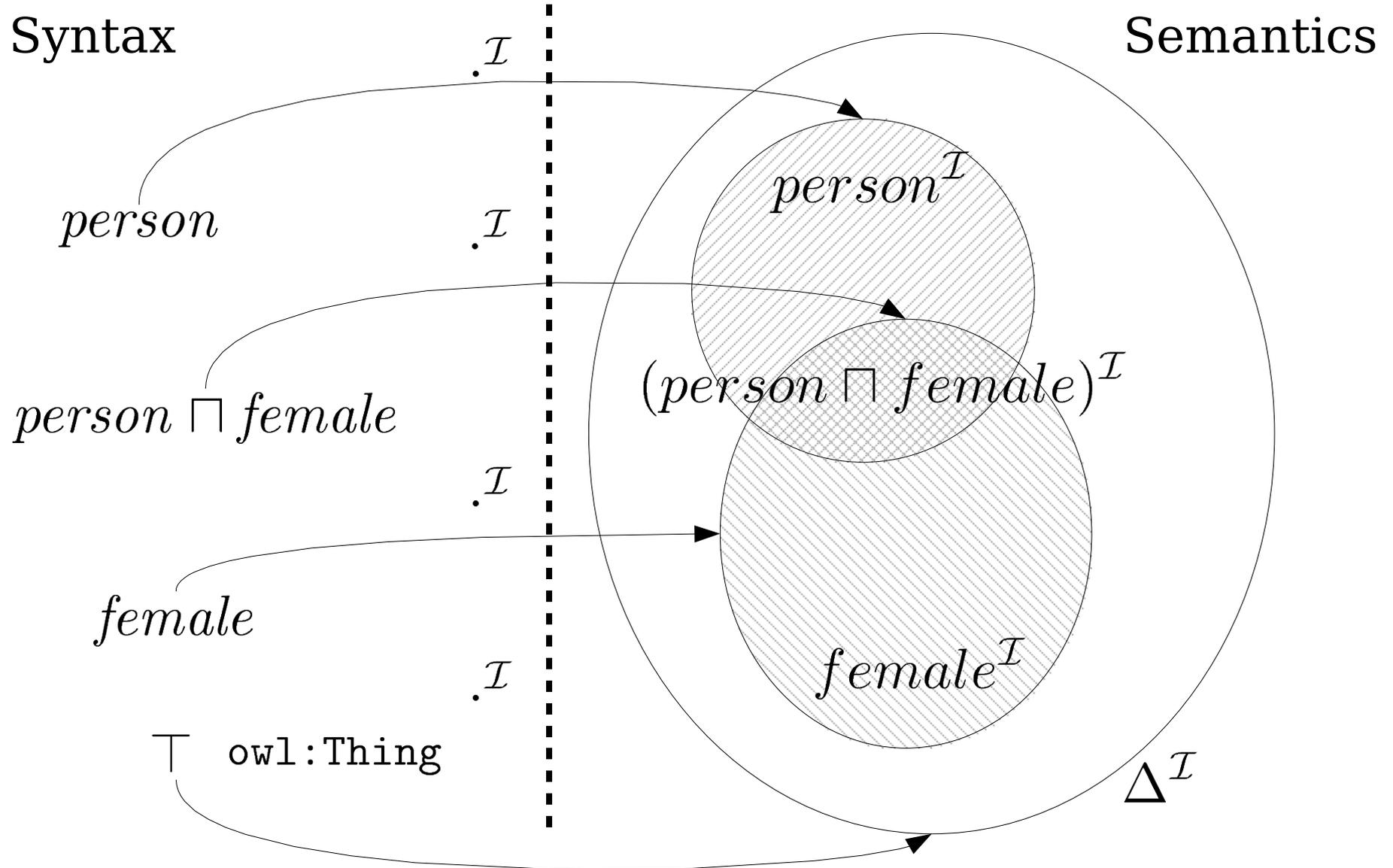
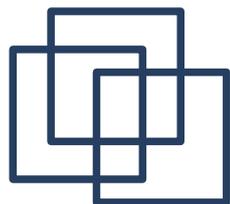


Illustration of Concept Semantics





Description Logics : Roles

- Represent relationships = sets of (binary) tuples
 - atomic roles : basic vocabulary, e.g. *has_child*
 - complex roles: e.g. *has_child*⁻¹
- Semantics via interpretation $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$
 - interpretation of a role = set of tuples from $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
 - function $\cdot^{\mathcal{I}}$ maps R to subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
 $has_child^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \quad \cdot^{\mathcal{I}} : \mathcal{N}_{\mathcal{R}} \mapsto 2^{\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}}$
- Role constructors, e.g. inverse role
 - constraint on interpretation of complex roles
 $(has_child^{-1})^{\mathcal{I}} = (has_child^{\mathcal{I}})^{-1}$

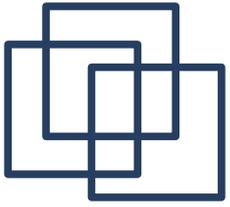
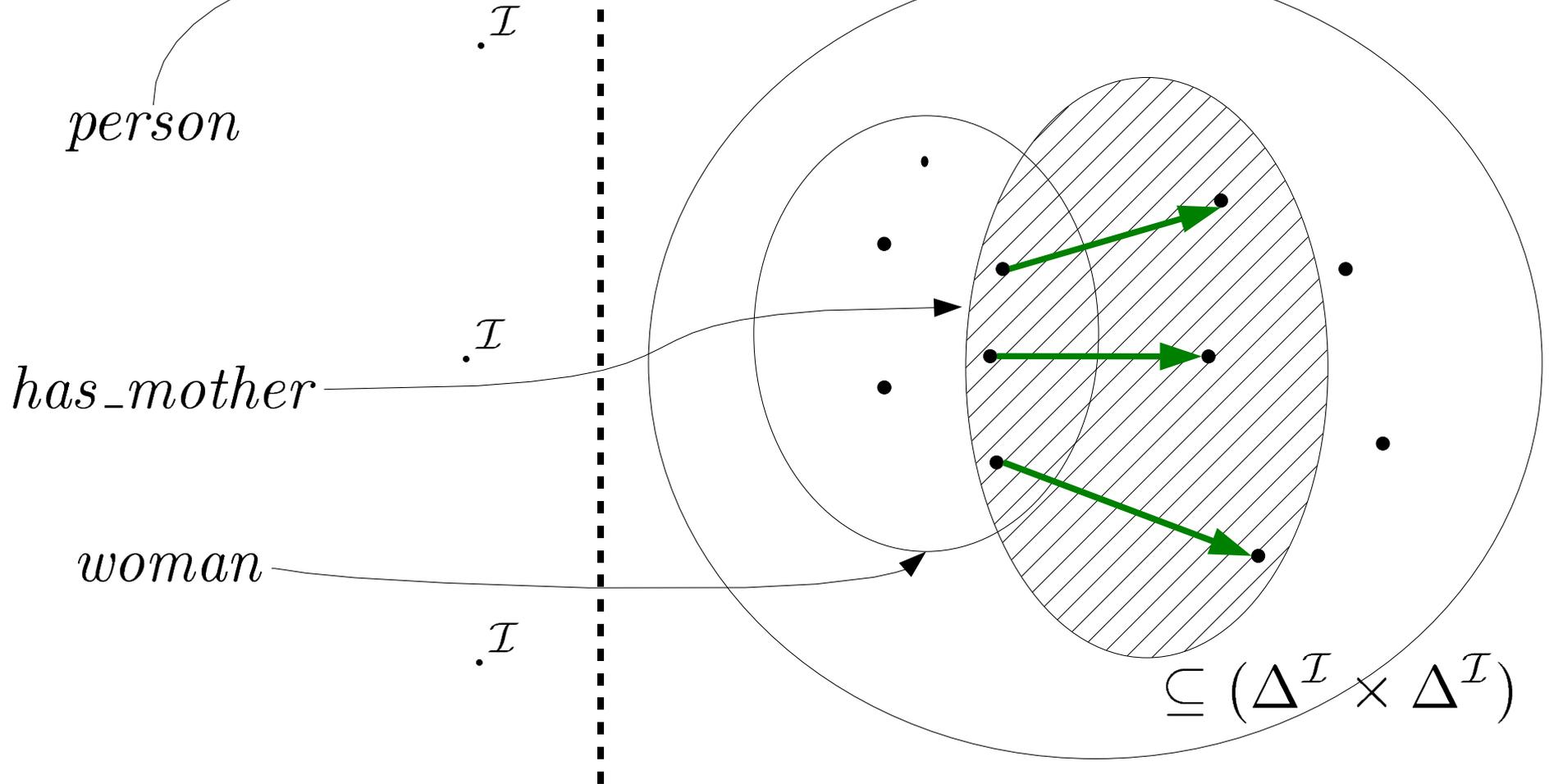
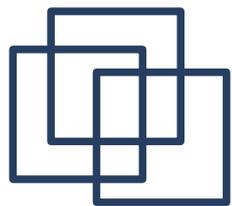


Illustration of Role Semantics

Syntax

Semantics





Concept Constructors : Conjunction

- DL syntax

$person \sqcap female$

- KRSS / Racer

(and person female)

- OWL RDF

```
<owl:Class>
```

```
  <owl:intersectionOf
```

```
    rdf:parseType="Collection">
```

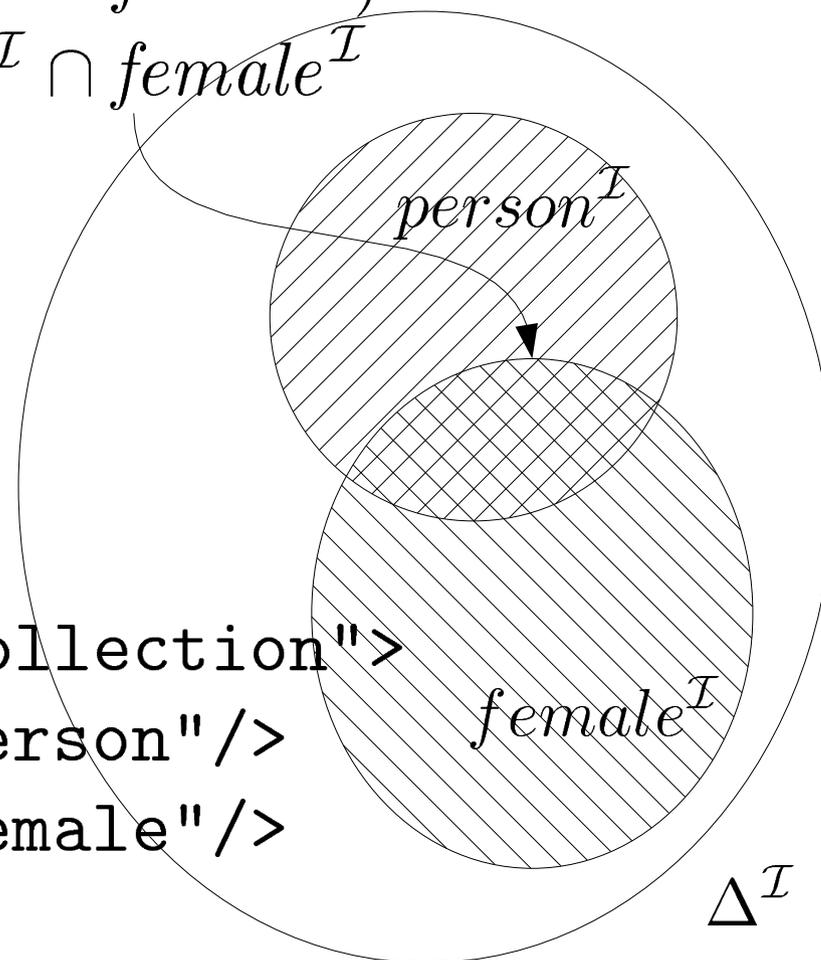
```
    <owl:Class rdf:ID="Person"/>
```

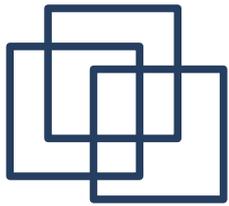
```
    <owl:Class rdf:ID="Female"/>
```

```
  </owl:intersectionOf>
```

```
</owl:Class>
```

$$(person \sqcap female)^{\mathcal{I}} = person^{\mathcal{I}} \cap female^{\mathcal{I}}$$





Concept Constructors : Disjunction

- DL syntax

$male \sqcup female$

- KRSS / Racer

(or male female)

- OWL RDF

```
<owl:Class>
```

```
  <owl:unionOf
```

```
    rdf:parseType="Collection">
```

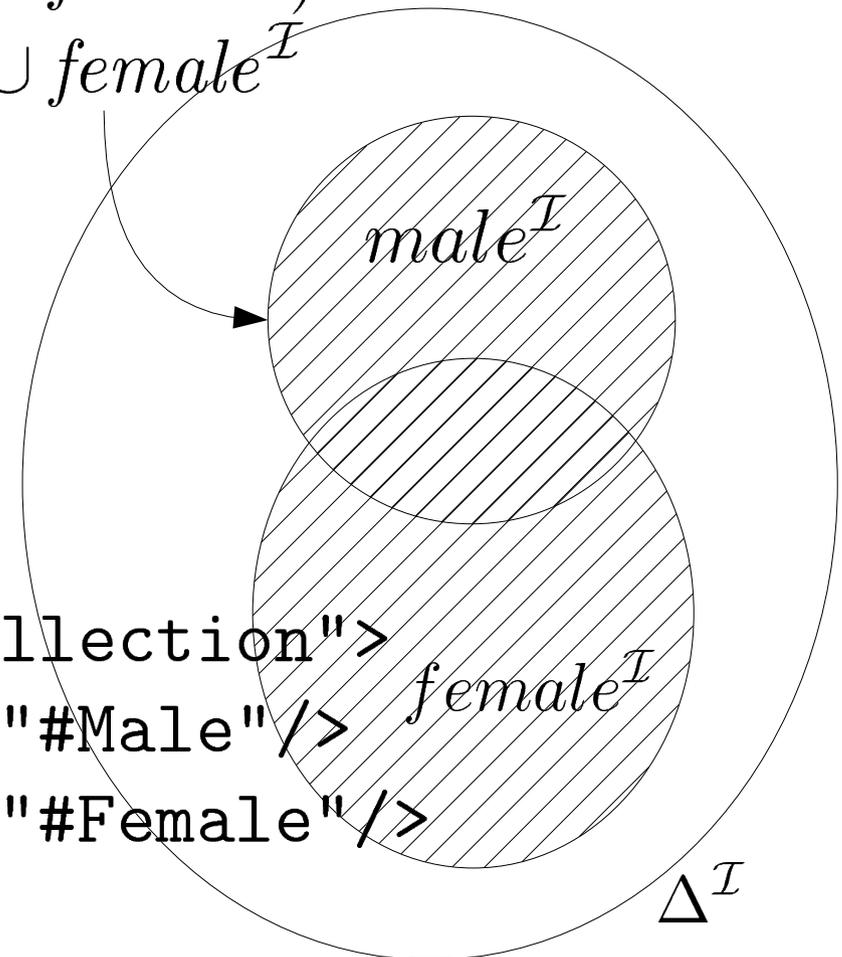
```
    <owl:Class rdf:about="#Male"/>
```

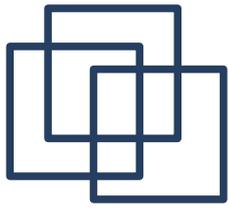
```
    <owl:Class rdf:about="#Female"/>
```

```
  </owl:unionOf>
```

```
</owl:Class>
```

$$(male \sqcup female)^{\mathcal{I}} = male^{\mathcal{I}} \cup female^{\mathcal{I}}$$





Concept Constructors : Negation

- DL syntax

$\neg female$

- KRSS / Racer

(not female)

- OWL RDF

```
<owl:Class>
```

```
<owl:complementOf
```

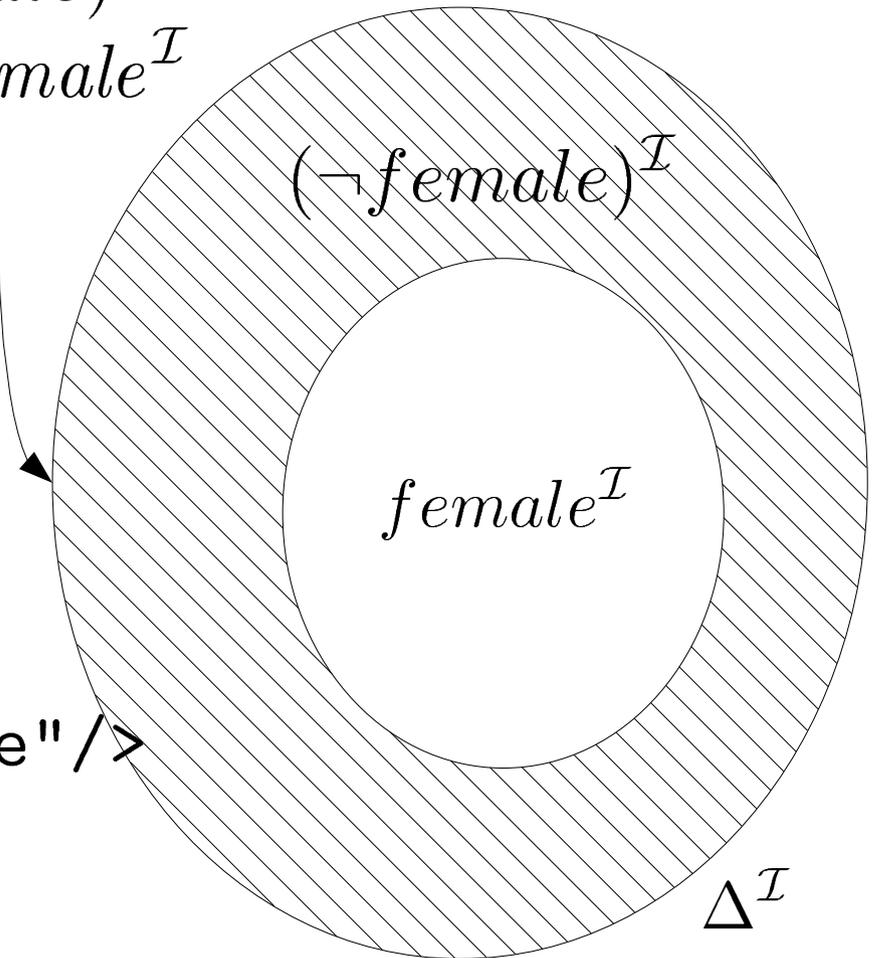
```
<owl:Class
```

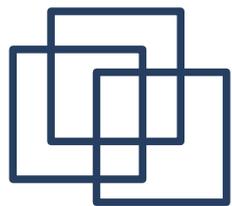
```
  rdf:about="#Female"/>
```

```
</owl:complementOf>
```

```
</owl:Class>
```

$$(\neg female)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus female^{\mathcal{I}}$$





Concept Constructors : Existentials

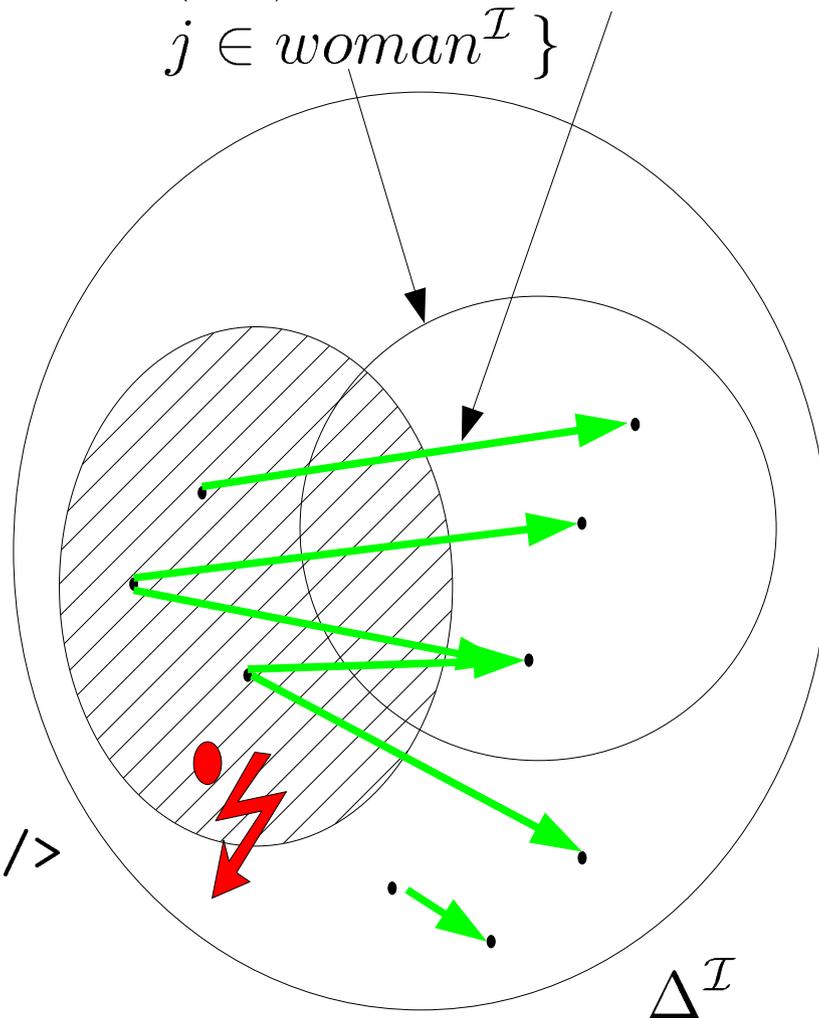
- DL syntax $(\exists has_mother.woman)^{\mathcal{I}} =$
 $\exists has_mother.woman \{ i \in \Delta^{\mathcal{I}} \mid \exists j : (i, j) \in has_mother^{\mathcal{I}}, j \in woman^{\mathcal{I}} \}$

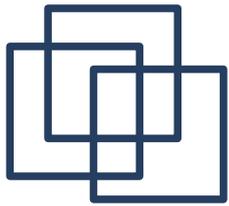
- KRSS / Racer

(some has-mother woman)

- OWL RDF

```
<owl:Restriction>  
  <owl:onProperty>  
    <owl:ObjectProperty  
      rdf:about="#hasMother"/>  
  </owl:onProperty>  
  <owl:someValuesFrom>  
    <owl:Class rdf:about="#Woman"/>  
  </owl:someValuesFrom>  
</owl:Restriction>
```





Concept Constructors : Universals

- DL syntax

$\forall has_mother.woman$

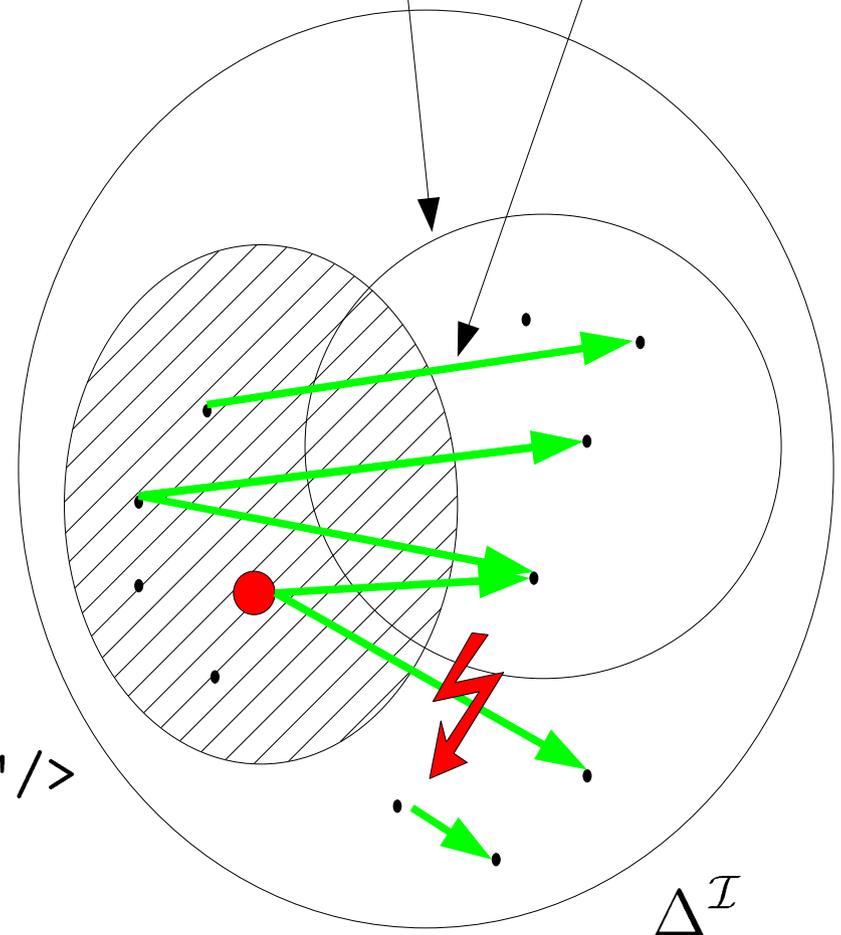
$$(\forall has_mother.woman)^{\mathcal{I}} = \{i \in \Delta^{\mathcal{I}} \mid \forall j : (i, j) \in has_mother^{\mathcal{I}} \rightarrow j \in woman^{\mathcal{I}}\}$$

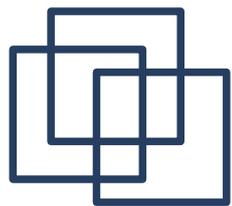
- KRSS / Racer

(all has-mother woman)

- OWL RDF

```
<owl:Restriction>
  <owl:onProperty>
    <owl:ObjectProperty
      rdf:about="#hasMother"/>
  </owl:onProperty>
  <owl:allValuesFrom>
    <owl:Class rdf:about="#Woman"/>
  </owl:allValuesFrom>
</owl:Restriction>
```



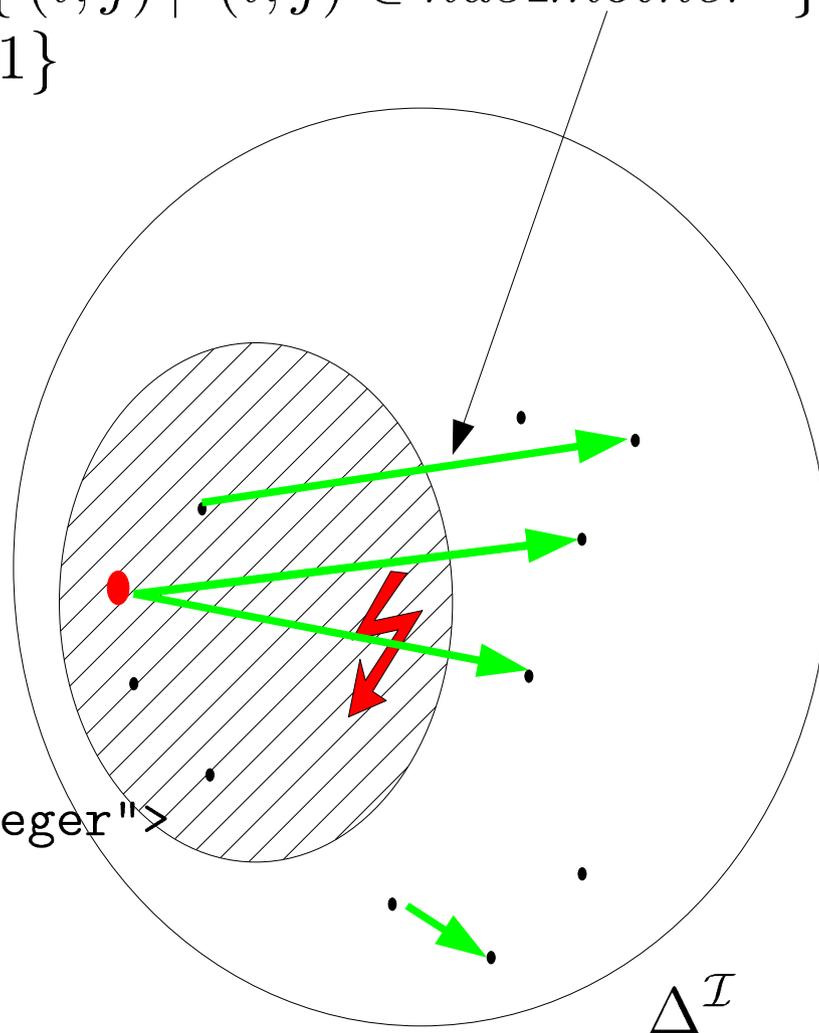


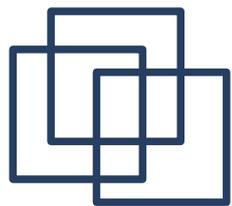
Constructors : Number Restrictions

- DL syntax $(\leq_1 \textit{has_mother})^{\mathcal{I}} =$
 $\leq_1 \textit{has_mother} \quad \{i \in \Delta^{\mathcal{I}} \mid \#\{(i, j) \mid (i, j) \in \textit{has_mother}^{\mathcal{I}}\} \leq 1\}$
- KRSS / Racer
(at-most 1 has-mother)

- OWL RDF

```
<owl:Restriction>  
  <owl:onProperty>  
    <owl:ObjectProperty  
      rdf:about="#hasMother"/>  
    </owl:onProperty>  
    <owl:maxCardinality  
      rdf:datatype="...#nonNegativeInteger">  
      1  
    </owl:maxCardinality>  
  </owl:Restriction>
```



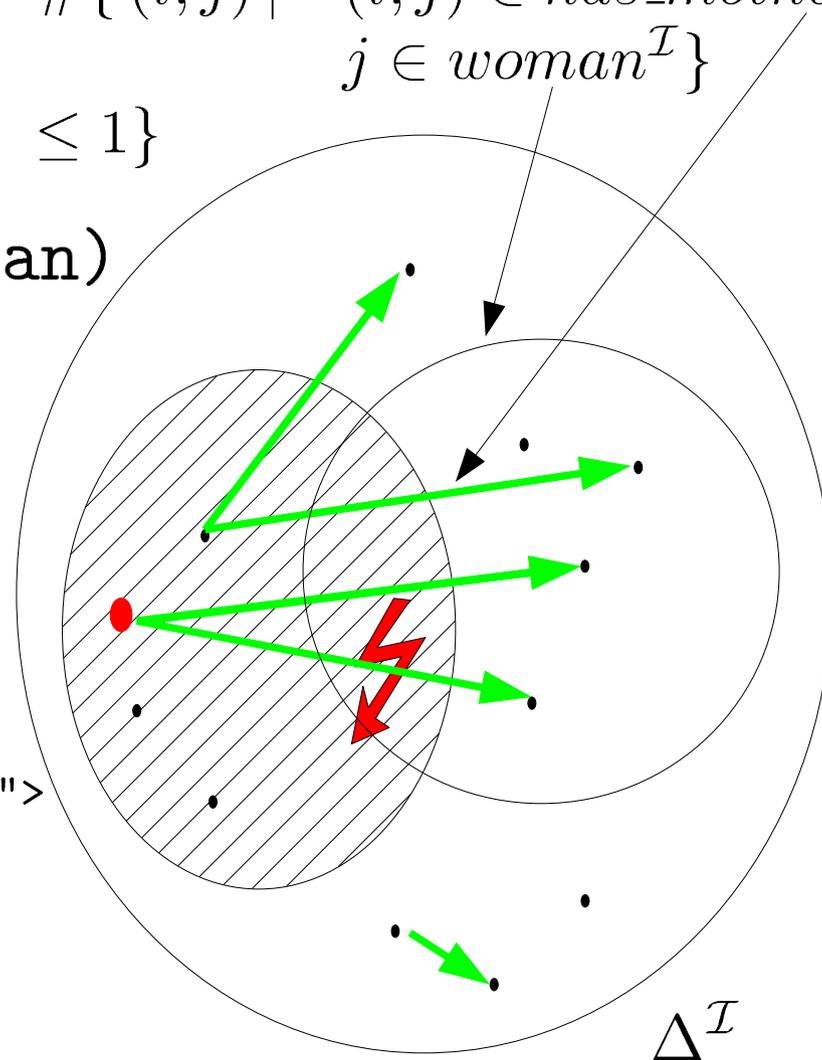


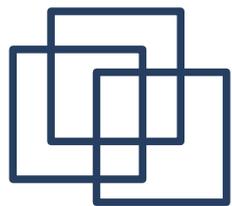
Constructors : Number Restrictions

- DL syntax $(\leq_1 \text{has_mother.woman})^{\mathcal{I}} = \{i \in \Delta^{\mathcal{I}} \mid \#\{(i, j) \mid (i, j) \in \text{has_mother}^{\mathcal{I}}, j \in \text{woman}^{\mathcal{I}}\} \leq 1\}$
 $\leq_1 \text{has_mother.woman}$
- KRSS / Racer $\leq 1\}$
(at-most 1 has-mother woman)

- OWL RDF

```
<owl:Restriction>  
<owl:onProperty>  
  <owl:ObjectProperty  
    rdf:about="#hasMother"/>  
</owl:onProperty>  
<owl:maxCardinality  
  rdf:datatype="...#nonNegativeInteger">  
  1  
</owl:maxCardinality>  
<owl2:onClass rdf:resource="#Woman"/>  
</owl:Restriction>
```



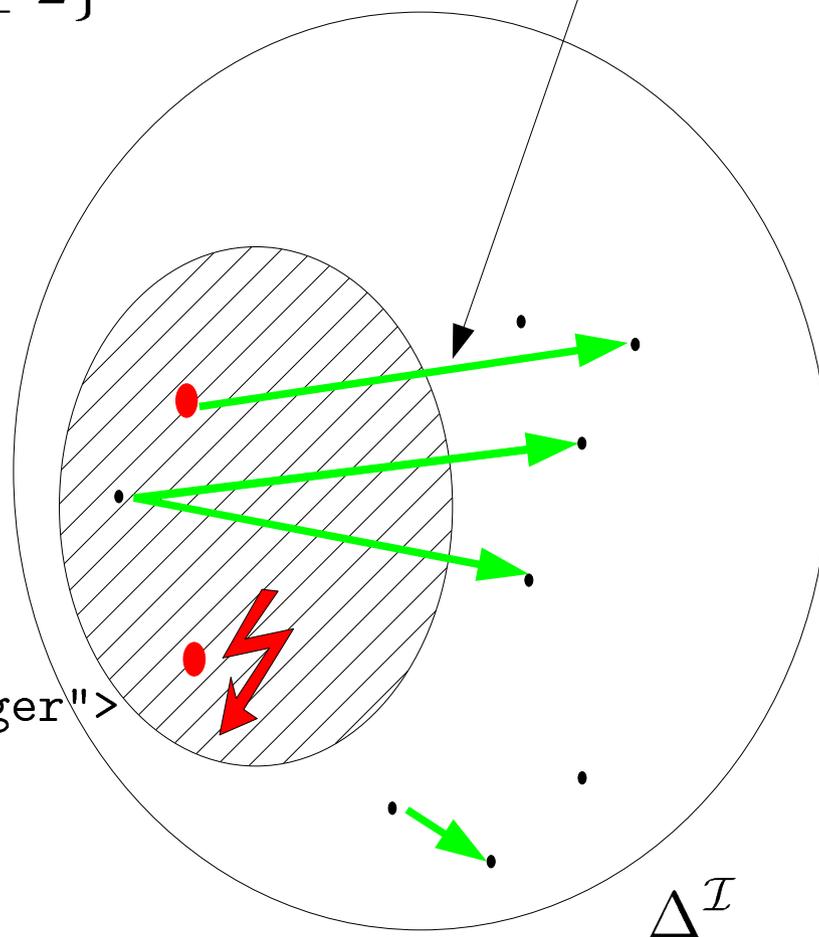


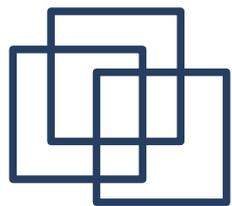
Constructors : Number Restrictions

- DL syntax $(\geq_2 \textit{has_child})^{\mathcal{I}} =$
 $\geq_2 \textit{has_child} \quad \{i \in \Delta^{\mathcal{I}} \mid \#\{(i, j) \mid (i, j) \in \textit{has_child}^{\mathcal{I}}\} \geq 2\}$
- KRSS / Racer
(at-least 2 has-child)

- OWL RDF

```
<owl:Restriction>
  <owl:onProperty>
    <owl:ObjectProperty
      rdf:about="#hasChild"/>
    </owl:onProperty>
    <owl:minCardinality
      rdf:datatype="...#nonNegativeInteger">
      2
    </owl:minCardinality>
  </owl:Restriction>
```





Constructors : Number Restrictions

- DL syntax

$\geq_2 \text{has_child.female}$

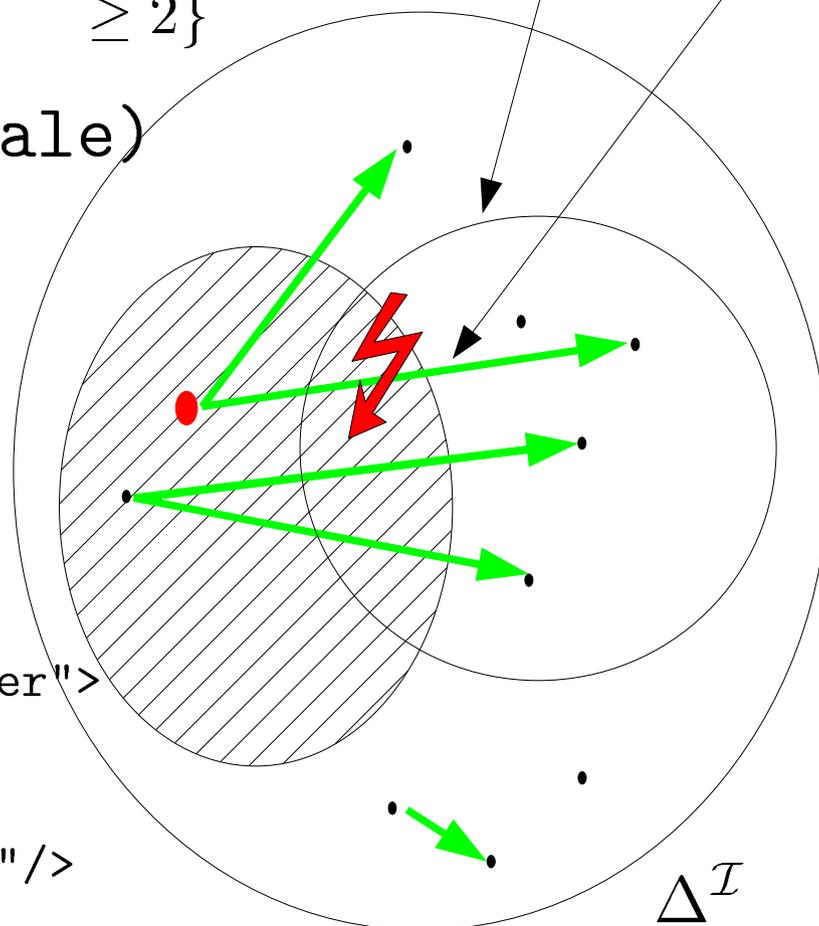
- KRSS / Racer

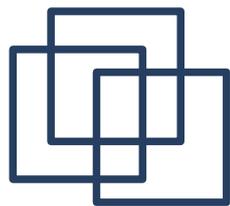
(at-least 2 has-child female)

- OWL RDF

```
<owl:Restriction>
  <owl:onProperty>
    <owl:ObjectProperty
      rdf:about="#hasChild"/>
    </owl:onProperty>
    <owl:minCardinality
      rdf:datatype="...#nonNegativeInteger">
      2
    </owl:minCardinality>
    <owl2:onClass rdf:resource="#Female"/>
  </owl:Restriction>
```

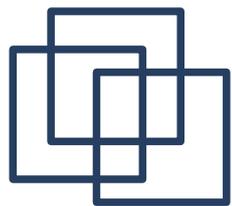
$$(\geq_2 \text{has_child.female})^{\mathcal{I}} = \{i \in \Delta^{\mathcal{I}} \mid \#\{(i, j) \mid (i, j) \in \text{has_child}^{\mathcal{I}}, j \in \text{female}^{\mathcal{I}}\} \geq 2\}$$





Inference Problems for Concepts

- Concept Satisfiability (Core Problem!)
 - exists some $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ such that $C^{\mathcal{I}} \neq \emptyset$?
 - then, $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}) \models C$ and $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is a model of C
- Concept Subsumption („Inheritance“)
 - does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold in all interpretations?
 - then, D subsumes C (subsumer / subsumee)
 - $\models C \sqsubseteq D$ iff $C \sqcap \neg D$ unsatisfiable
- Equivalence: $\models C \sqsubseteq D, \models D \sqsubseteq C$
- Disjointness
 - holds $C^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset$ in all interpretations?
 - iff $C \sqcap D$ unsatisfiable



Description Logics : TBox Axioms

- Constrain interpretations of (atomic) concepts

- enforce subset relationships

$$\mathcal{I} \models mother \sqsubseteq parent$$

$$\text{iff}$$
$$mother^{\mathcal{I}} \subseteq parent^{\mathcal{I}}$$

- enforce equivalences

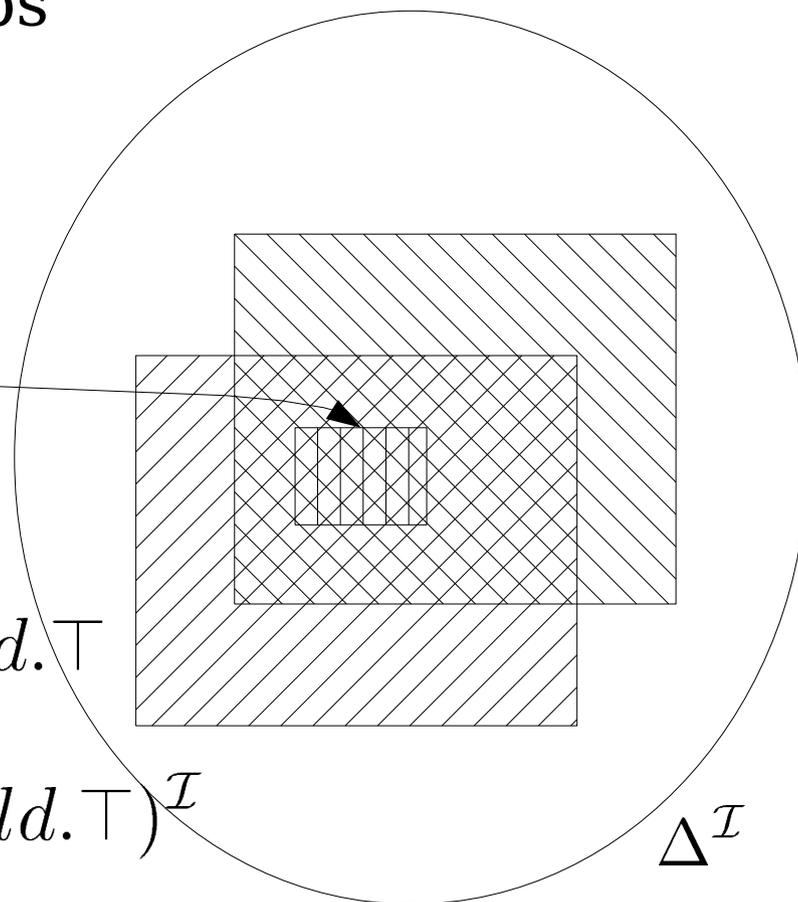
(„definitions“)

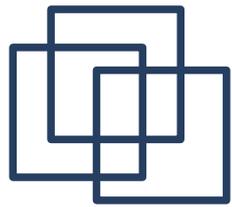
$$\mathcal{I} \models parent \doteq person \sqcap \exists has_child. \top$$

iff

$$parent^{\mathcal{I}} = person^{\mathcal{I}} \cap (\exists has_child. \top)^{\mathcal{I}}$$

- Nowadays, arbitrary concepts in axioms (GCIs)





Description Logics : TBox Axioms (2)

- DL Syntax

$mother \sqsubseteq parent$

- DL Syntax

$parent \equiv person \sqcap \exists has_child. \top$

- KRSS / Racer

(implies mother parent)

(define-primitive-concept
mother parent)

- KRSS / Racer

(equivalent parent
(and person ...))

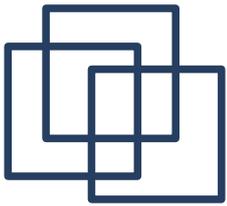
(define-concept parent
(and person ...))

- OWL

```
<owl:Class rdf:about="Mother">  
  <rdfs:subClassOf>  
    <owl:Class rdf:about="Parent"/>  
  </rdfs:subClassOf>  
</owl:Class>
```

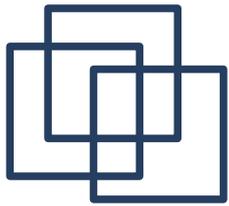
- OWL

```
<owl:Class rdf:about="Parent">  
  <owl:equivalentClass>  
    <owl:Class rdf:about="Person"/>  
  <owl:Class>  
    <owl:intersectionOf>  
      ...  
    </owl:intersectionOf>  
  </owl:Class>  
</owl:equivalentClass>  
</owl:Class>
```



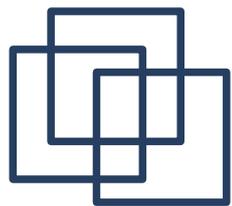
DLs as First Order Logic

- **Concepts:** FOPL formulas with one free variable
 $person(x) \wedge \exists y.has_child(x, y)$
- **Roles:** binary atoms with two free variables
 $has_child(x, y)$
- **Individuals:** constants
 $betty$
- **Axioms**
 $\forall x.(parent(x) \leftrightarrow person(x) \wedge \exists y.has_child(x, y))$
 $\forall x.(mother(x) \rightarrow parent(x))$
 $\forall x, y.(has_child(x, y) \leftrightarrow has_child(y, x))$
 $\forall x, y, z.(has_descendant(x, y) \wedge has_descendant(y, z) \rightarrow has_descendant(x, z))$



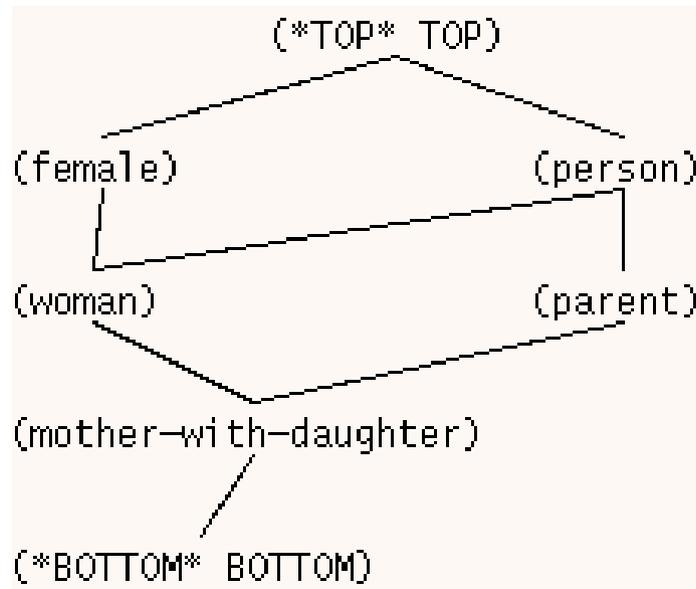
Inference Problems for TBoxes

- Concept satisfiability (disjointness, subsumption, equivalence) w.r.t. a Tbox, e.g.
 - $parent \sqcap \neg mother$ unsat. w.r.t. TBox
 - $woman \sqcap \exists has_child.female \sqsubseteq parent$ due to TBox
- Reasoning example:
 - $woman \sqcap \exists has_child.female \sqsubseteq parent$ iff
 - $woman \sqcap \exists has_child.female \sqcap \neg parent$ unsat. iff
 - $woman \sqcap \exists has_child.female \sqcap \neg (person \sqcap \exists has_child.\top)$ iff
 - $person \sqcap \dots \sqcap \exists has_child.female \sqcap \neg person$ unsat. AND
 - $person \sqcap \dots \sqcap \exists has_child.female \sqcap \forall has_child.\perp$ unsat.
- ... only that simple for simple (unfoldable) TBoxes



Inference Problems for TBoxes (2)

- TBox coherence check
 - unsatisfiable concept names or roles other than \perp ? e.g.
 $C \sqsubseteq D, D \sqsubseteq \neg C, \dots$



- TBox satisfiability
 - all concepts names unsatisfiable? e.g.
 $C \sqsupseteq D, D \sqsupseteq \neg C, \dots$
- Taxonomy computation
 - compute most specific subsumers and most general subsumees for names

$woman \sqsupseteq$

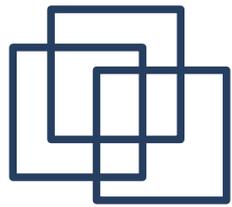
$person \sqcap female$

$parent \sqsupseteq$

$person \sqcap \exists has_child. \top$

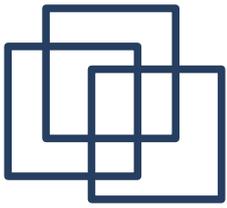
$mother_w_daughter \sqsupseteq$

$woman \sqcap \exists has_child. female$



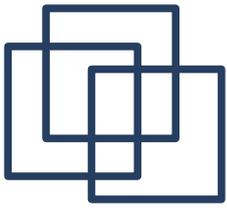
More Tbox Axioms: Role Declarations

- Sub / super roles $has_mother \dot{\sqsubseteq} has_parent$
- Transitive roles $transitive(has_descendant)$
 - no number restrictions f. trans. Roles (or roles with trans. subroles) allowed!
- Functional roles $functional(has_mother)$
- Being inverses $has_parent \dot{=} has_child^{-1}$
- Domain & range restrictions
 - $domain(has_mother) = person$
 $\exists has_mother. \top \dot{\sqsubseteq} person$
 - $range(has_mother) = mother$
 $\top \dot{\sqsubseteq} \forall has_mother. mother$



Syntax of Role Declarations

```
(define-primitive-role
  has-descendant
  :transitive t)
<owl:TransitiveProperty
  rdf:about="has-descendant"/>
<owl:ObjectProperty rdf:about="has-child">
  <rdfs:subPropertyOf
    rdf:resource="has-descendant"/>
  <owl:inverseOf rdf:resource="has-parent"/>
</owl:ObjectProperty>
(define-primitive-role
  has-child
  :parent has-descendant)
<owl:ObjectProperty rdf:about="has-mother">
  <rdfs:subPropertyOf rdf:resource="has-parent"/>
  <rdfs:domain>
    <owl:Class>
      <owl:intersectionOf ...>
        <owl:Class rdf:about="person"/>
      </owl:intersectionOf>
    </owl:Class>
  </rdfs:domain>
  <rdfs:range>
    <owl:Class rdf:about="mother"/>
  </rdfs:range>
</owl:ObjectProperty>
(define-primitive-role
  has-mother
  :parent has-parent
  :domain person
  :range mother
  :feature t)
<owl:FunctionalProperty rdf:about="has-mother"/>
```

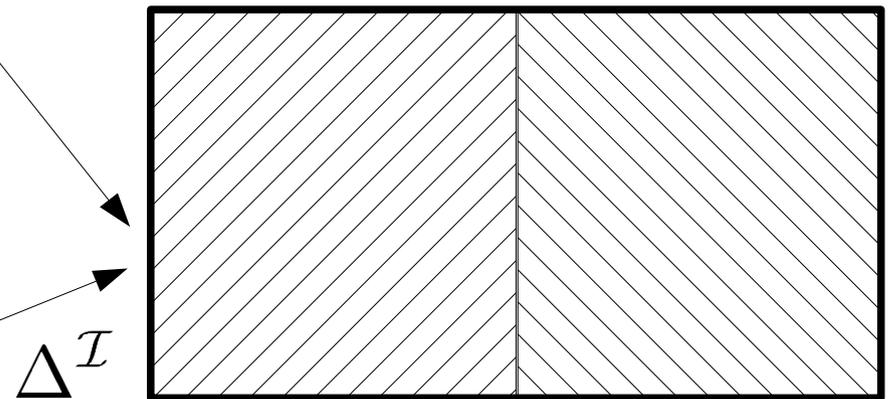


TBox Patterns : Covering Axioms

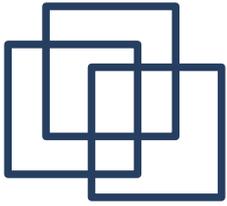
- Disjointness $plant \dot{\sqsubseteq} \neg animal (= animal \dot{\sqsubseteq} \neg plant)$
 - OWL axiom `<owl:disjointWith>`

- Covering axioms $living_thing \dot{\sqsupseteq} plant \sqcup animal$
 - OWL axiom

`<owl:disjointUnion>`

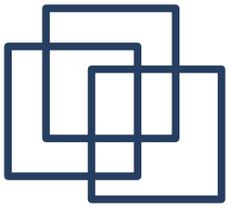


- Global axioms $\top \dot{\sqsubseteq} plant \sqcup animal$
- Global consistency condition $plant \sqcap animal \dot{\sqsubseteq} \perp$

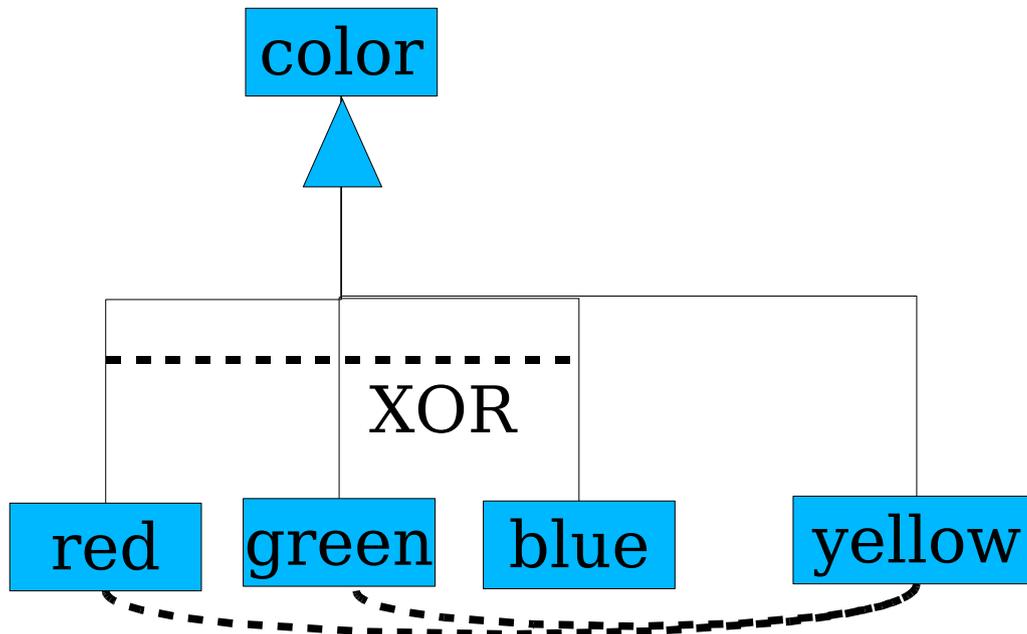


Implicit Subsumptions

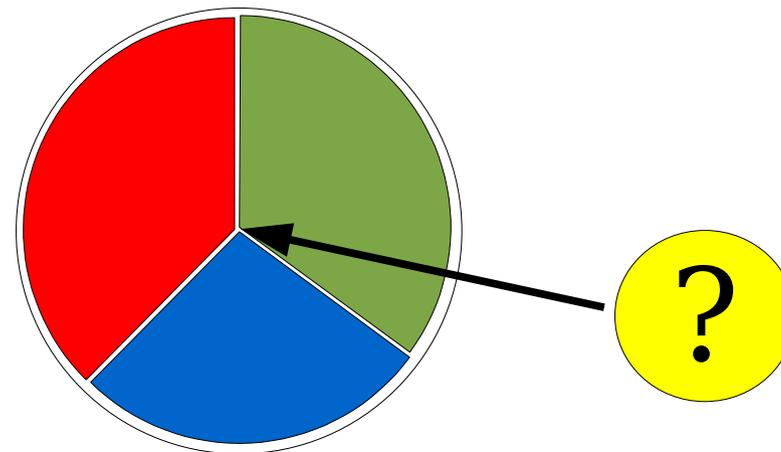
- In an axiom $D \sqsubseteq X$
 - D is **sufficient** for X („given D , X follows“)
 - X is **necessary** for D („without X , D cannot hold“)
 - Since $\neg X \sqsubseteq \neg D$, also $\neg X$ is sufficient for $\neg D$
 - no sufficient conditions for D !
 - But: $D \doteq X \sqcap D^*$ for some fresh D^*
- However, $C \sqsubseteq D$ can never hold, since
$$C \sqcap \neg D \text{ unsat. iff } C \sqcap \neg(X \sqcap D^*) \text{ unsat. iff}$$
$$(C \sqcap \neg X) \text{ unsat. AND } (C \sqcap \neg D^*) \text{ unsat.}$$
the latter can never happen, since D^* was fresh
- Thus: no **implicit** subsumption without proper sufficient conditions for subsumer (D)

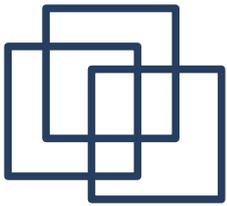


Implicit Subsumptions (2)

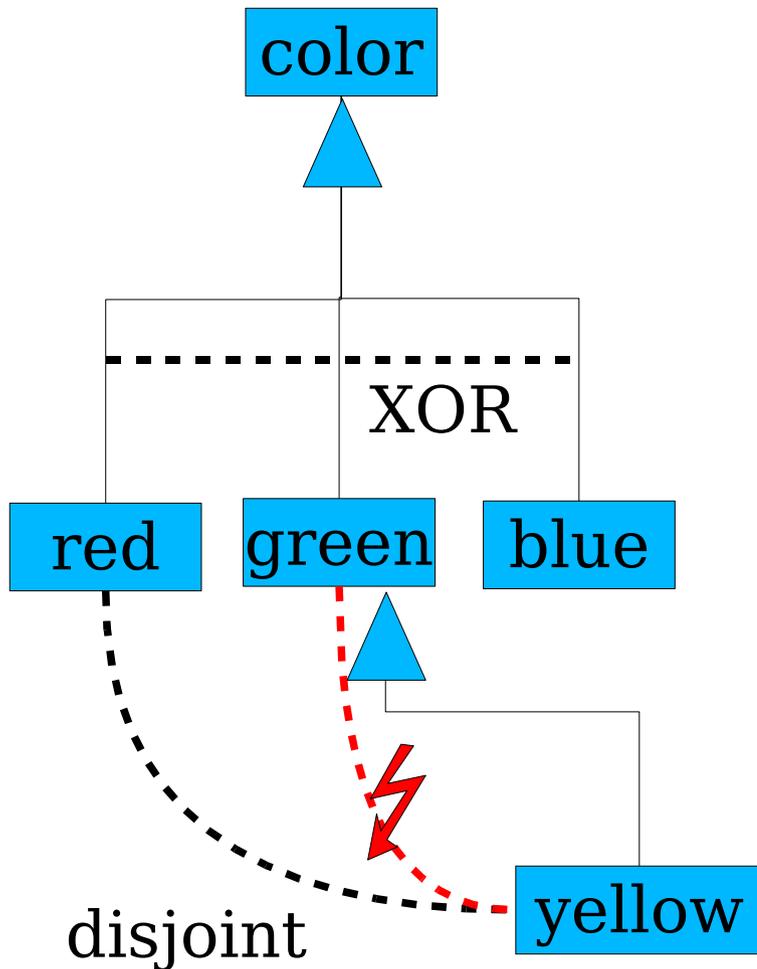


$color \dot{\sqsubseteq} red \sqcup green \sqcup blue$
 $red \dot{\sqsubseteq} \neg green$
 $red \dot{\sqsubseteq} \neg blue$
 $green \dot{\sqsubseteq} \neg blue$
 $yellow \dot{\sqsubseteq} color$
 $yellow \dot{\sqsubseteq} \neg green$
 $yellow \dot{\sqsubseteq} \neg red$

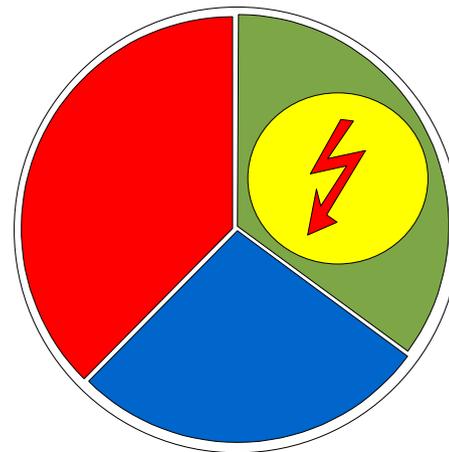


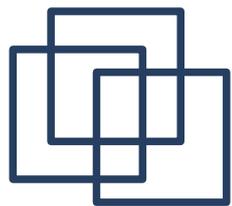


Implicit Subsumptions (3)

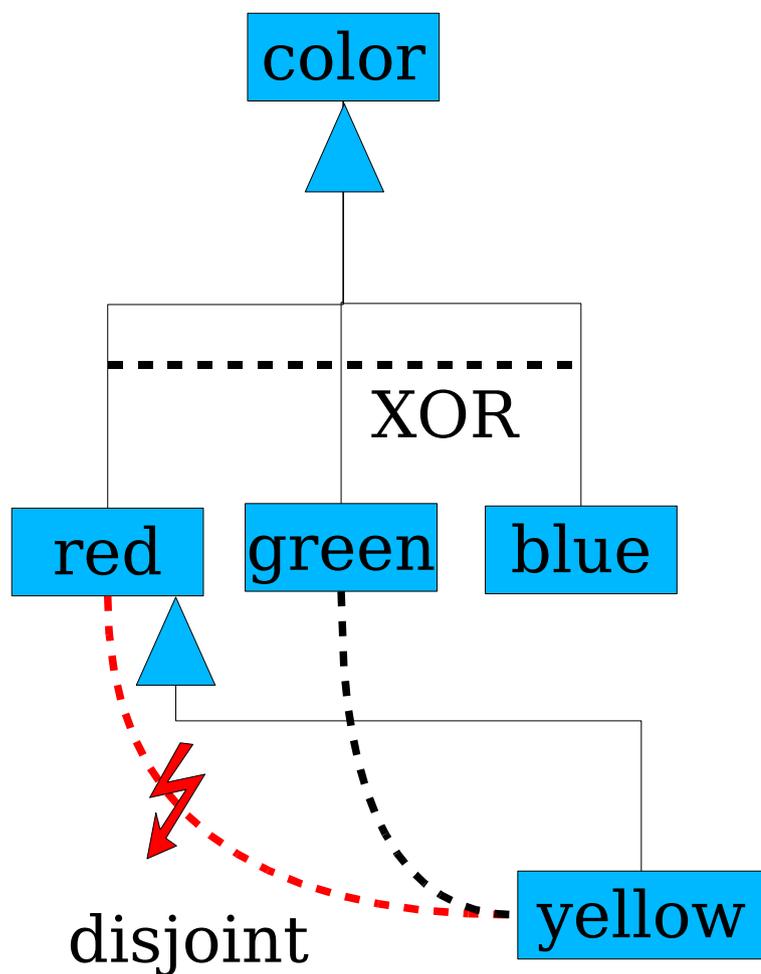


$color \dot{\sqsubseteq} red \sqcup green \sqcup blue$
 $red \dot{\sqsubseteq} \neg green$
 $red \dot{\sqsubseteq} \neg blue$
 $green \dot{\sqsubseteq} \neg blue$
 $yellow \dot{\sqsubseteq} color$
 $yellow \dot{\sqsubseteq} \neg green$
 $yellow \dot{\sqsubseteq} \neg red$

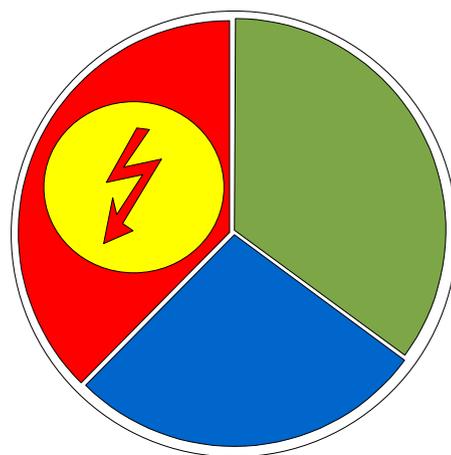


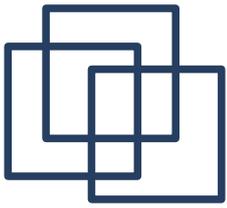


Implicit Subsumptions (4)

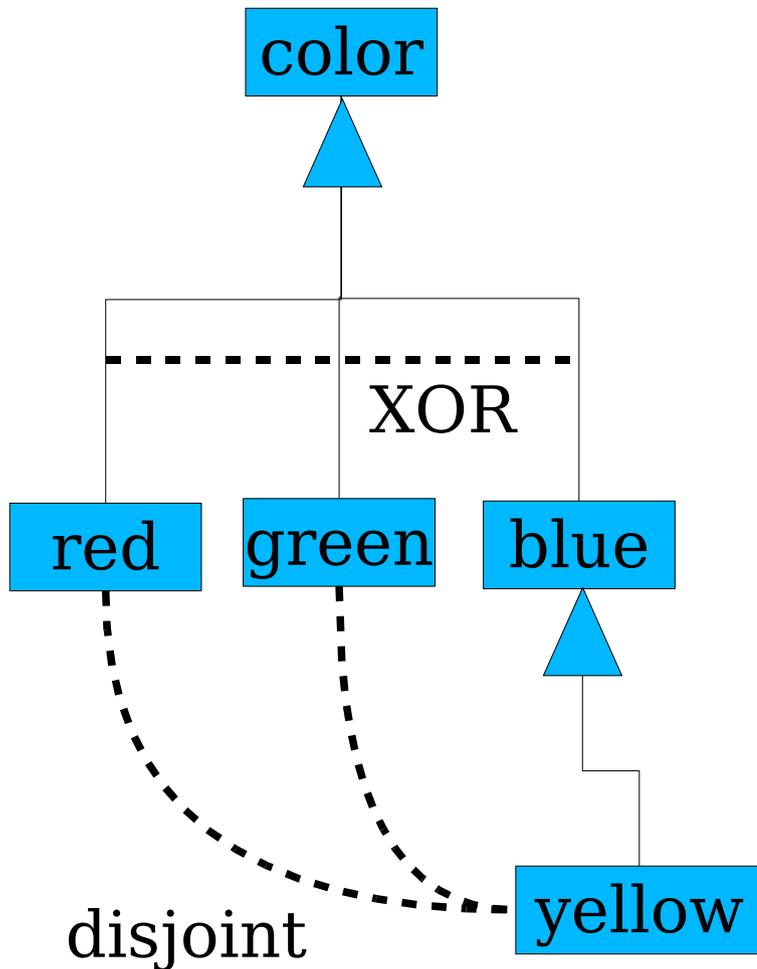


$color \dot{\sqsubseteq} red \sqcup green \sqcup blue$
 $red \dot{\sqsubseteq} \neg green$
 $red \dot{\sqsubseteq} \neg blue$
 $green \dot{\sqsubseteq} \neg blue$
 $yellow \dot{\sqsubseteq} color$
 $yellow \dot{\sqsubseteq} \neg green$
 $yellow \dot{\sqsubseteq} \neg red$

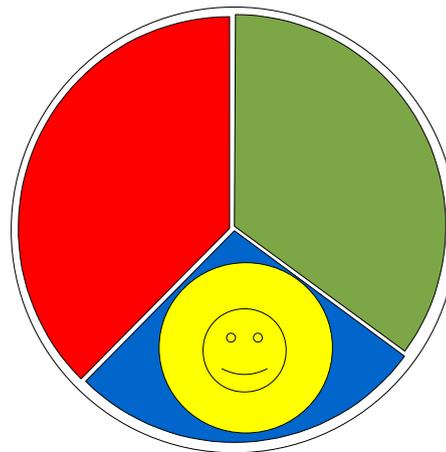


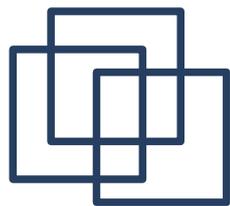


Implicit Subsumptions (5)

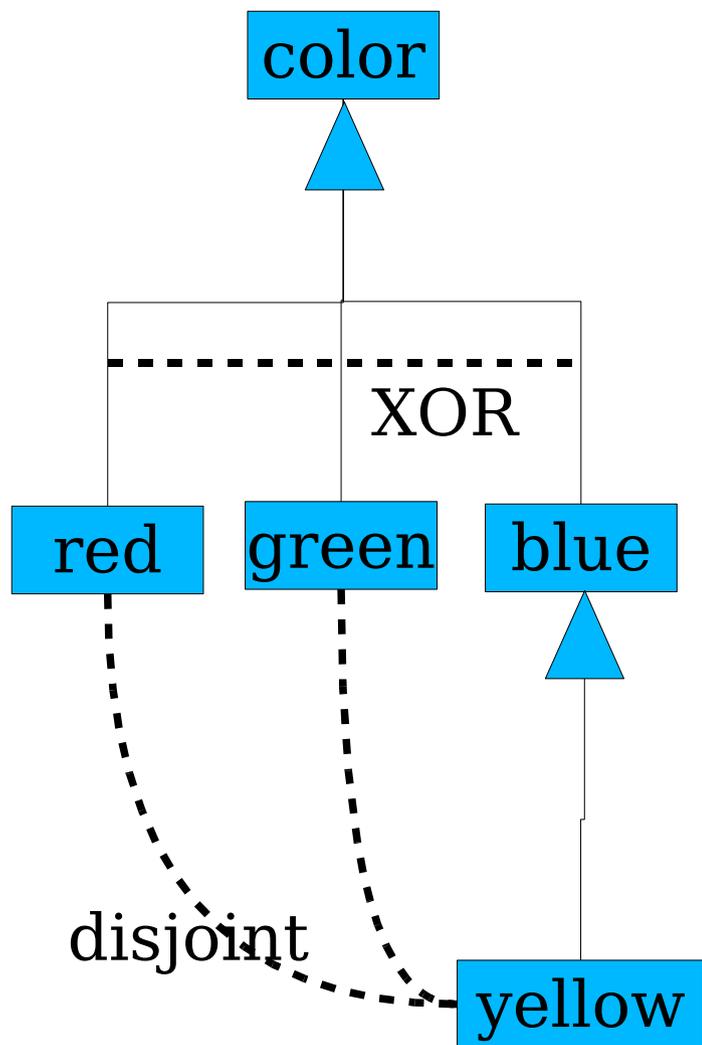


$color \dot{\sqsubseteq} red \sqcup green \sqcup blue$
 $red \dot{\sqsubseteq} \neg green$
 $red \dot{\sqsubseteq} \neg blue$
 $green \dot{\sqsubseteq} \neg blue$
 $yellow \dot{\sqsubseteq} color$
 $yellow \dot{\sqsubseteq} \neg green$
 $yellow \dot{\sqsubseteq} \neg red$





Implicit Subsumption Relationships



$$color \dot{\sqsubseteq} red \sqcup green \sqcup blue$$

$$red \dot{\sqsubseteq} \neg green$$

$$red \dot{\sqsubseteq} \neg blue$$

$$green \dot{\sqsubseteq} \neg blue$$

$$yellow \dot{\sqsubseteq} color$$

$$yellow \dot{\sqsubseteq} \neg green$$

$$yellow \dot{\sqsubseteq} \neg red$$

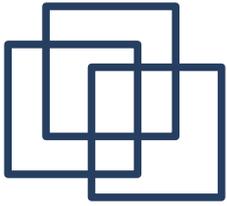
$$yellow \sqcap \neg blue \models$$

$$yellow \sqcap \neg green \sqcap \neg red \sqcap color \sqcap \neg blue \models$$

$$yellow \sqcap \neg green \sqcap \neg red \sqcap \neg blue \sqcap$$

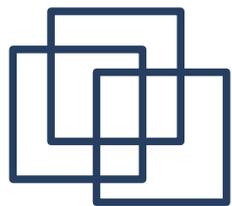
$$(red \sqcup green \sqcup blue) \models$$

$$yellow \sqcap \neg green \sqcap \neg red \sqcap \neg blue \sqcap blue \models \perp$$



DL Naming Schema

DL	Expressive Means
\mathcal{ALC}	$\sqcap, \sqcup, \exists R.C, \forall R.C$
$S(\mathcal{ALC}_{R^+})$	$\mathcal{ALC} + \mathcal{R}^+$ for transitively closed roles
\mathcal{ALCI}	$\mathcal{ALC} + \mathcal{I}$ for inverse roles
\mathcal{ALCH}	$\mathcal{ALC} + \mathcal{H}$ for role hierarchies
\mathcal{ALCN}	$\mathcal{ALC} + \mathcal{N}$ for number restrictions
\mathcal{ALCQ}	$\mathcal{ALC} + \mathcal{Q}$ for qualified number restrictions
\mathcal{ALCO}	$\mathcal{ALC} + \mathcal{O}$ for nominals
OWL = $\mathcal{SHOIN}(\mathcal{D}^-)$	\mathcal{D}^- for datatypes
RACERPRO = $\mathcal{SHIQ}(\mathcal{D}^-)$	\mathcal{D}^- for concrete domains
OWL2 = $\mathcal{SROIQ}(\mathcal{D}^+)$	\mathcal{R} for complex role inclusions



Individuals and Relationships: ABox

- Abox = set of ABox assertions (axioms)
- Instance and role assertions (plus same-as, different-from, ...)

$\{betty : person, (betty, charles) : has_child\}$

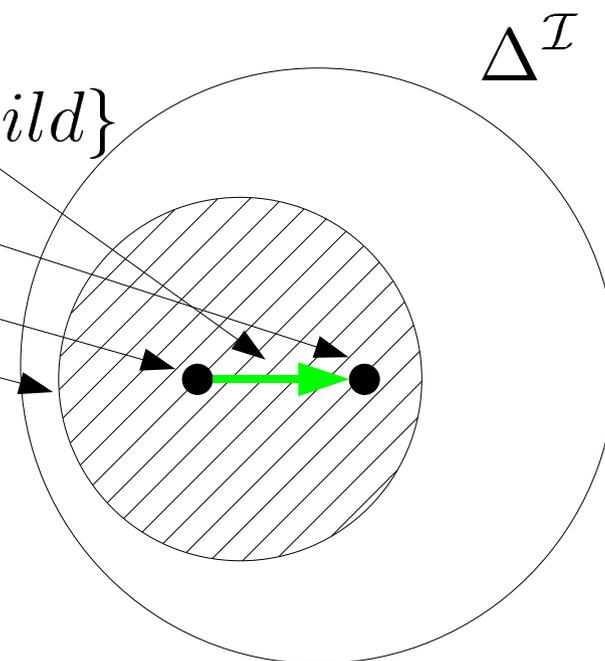
(instance betty person)

(related betty charles has-child)

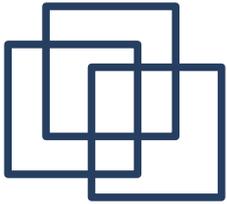
```
<Person rdf:ID="bettty">
```

```
  <hasChild rdf:resource="#charles"/>
```

```
</Person>
```

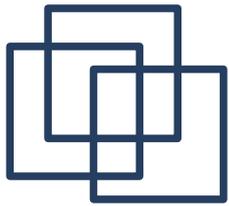


- $\cdot^{\mathcal{I}}$ maps individuals to elements in $\Delta^{\mathcal{I}}$
 - $\mathcal{I} \models betty : person$ iff $betty^{\mathcal{I}} \in person^{\mathcal{I}}$
 - $\mathcal{I} \models (betty, charles) : has_child$ iff $(betty^{\mathcal{I}}, charles^{\mathcal{I}}) \in has_child^{\mathcal{I}}$



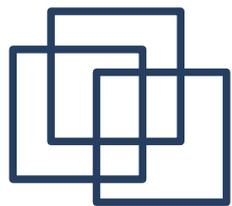
ABox Inference Services

- Abox satisfiability (w.r.t. a possibly empty TBox)
 - does the Abox have a model?
 $\{betty : \neg parent, betty : person, (betty, charles) : has_child\}$
- Individual realization
 - compute the (most specific) concept names an individual is an instance of, e.g. in
 $\{betty : person, (betty, charles) : has_child\}$
it is realized that *betty* is an instance of *parent*
- Instance checking: is *betty* an instance of *parent*?
- Role filler checking: is *charles* a filler (successor) of the *has_child* role of *betty* ?

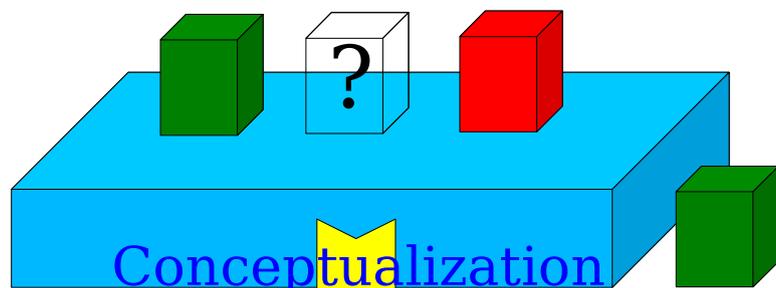


Abox Inference Services (2)

- Abox retrieval services
 - Instance retrieval
 $\text{concept_instances}(\textit{parent}) = \{\textit{betty}\}$
 $(\text{concept-instances } \textit{parent}) \rightarrow (\textit{betty})$
 - Role filler retrieval
 $\text{role_fillers}(\textit{betty}, \textit{has_child}) = \{\textit{charles}\}$
 $(\text{individual-fillers } \textit{betty} \textit{ has-child}) \rightarrow (\textit{charles})$
 - ... and some more
 - Recent research focus: **ABox query answering**
 - RDF QLs: SPARQL, RQL, ...
 - „true“ DL QLs: nRQL, OWLQL, ...



Abox Query Answering

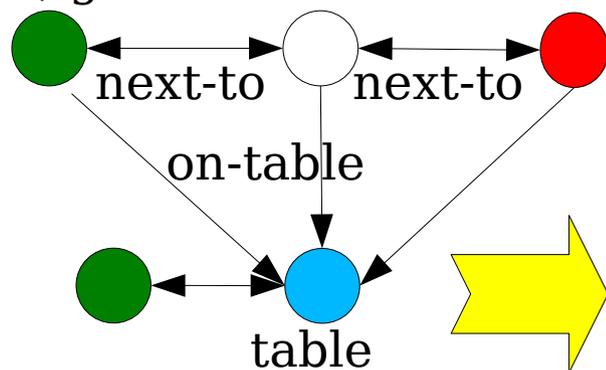


Conceptualization
(Abstraction)

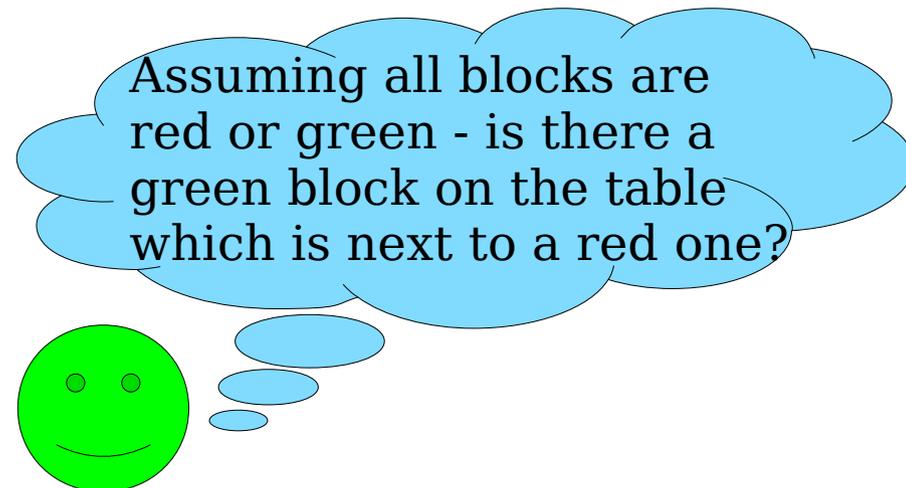
on-table, next-to, block, green, red, ...

Formalization

block, green



Problem Solving

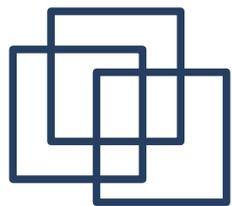


$$\{ \text{block} \sqsubseteq \text{red} \sqcup \text{green}, \text{next_to} \doteq \text{next_to}^{-1} \}$$

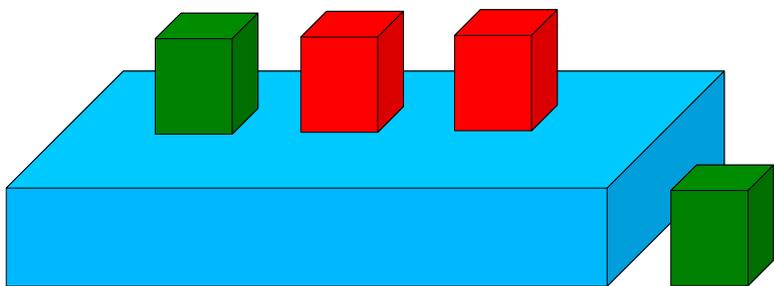
$$\{ t : \text{table}, lb : \text{green} \sqcap \text{block}, rb : \text{red} \sqcap \text{block}, \\ mb : \text{block}, ob : \text{green} \sqcap \text{block}, \\ (lb, t) : \text{on_table}, (ml, t) : \text{on_table}, (rb, t) : \text{on_table}, \\ (gb, ob) : \text{next_to}, (ob, rb) : \text{next_to} \}$$

Ask for instances of the concept

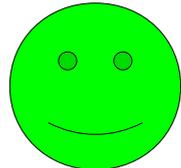
$$\text{table} \sqcap \\ \exists \text{on_table}^{-1}. (\text{block} \sqcap \text{green} \sqcap \\ \exists \text{next_to}. (\text{red} \sqcap \text{block}))$$



Abox Query Answering (2)

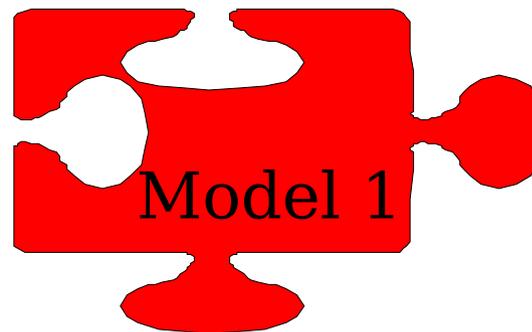
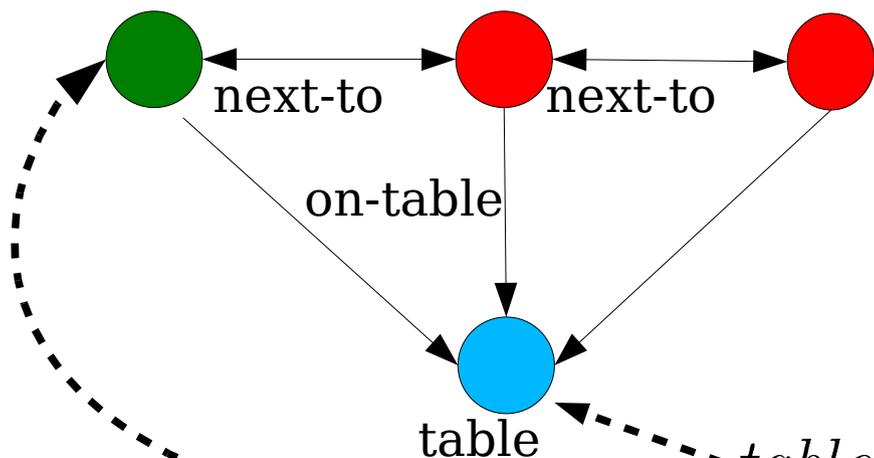


There are two possibilities. If the middle block is red, then the green left block is next to a red one. But...



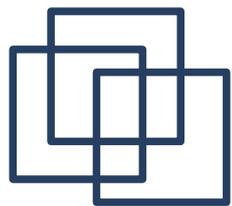
block, green

block, red

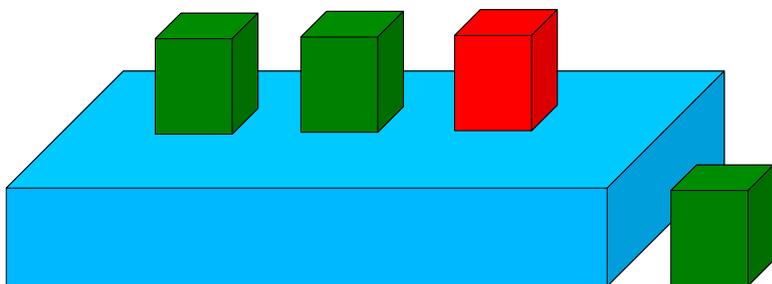


$table \sqsupset$

$$\exists on_table^{-1}. (block \sqsupset green \sqsupset \exists next_to.(red \sqsupset block))$$



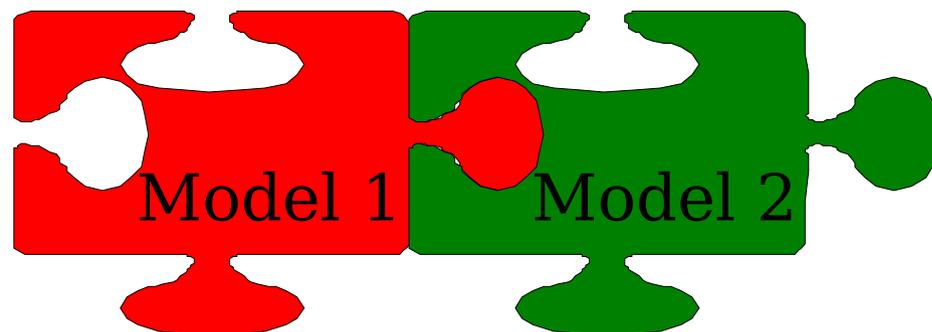
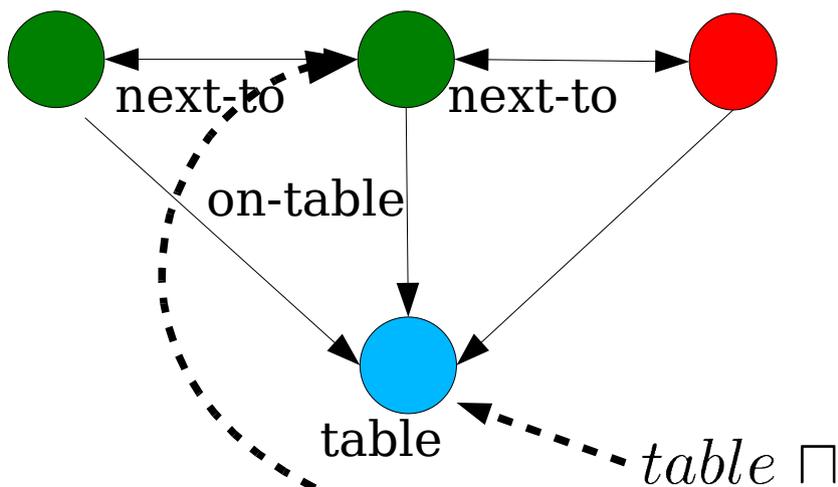
Abox Query Answering (3)



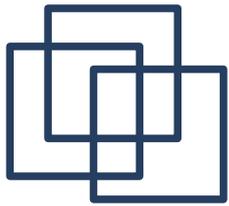
... if the middle block is green, then it is also next to the right Block, which is red. So, yes, there ALWAYS EXISTS such a block on the table!



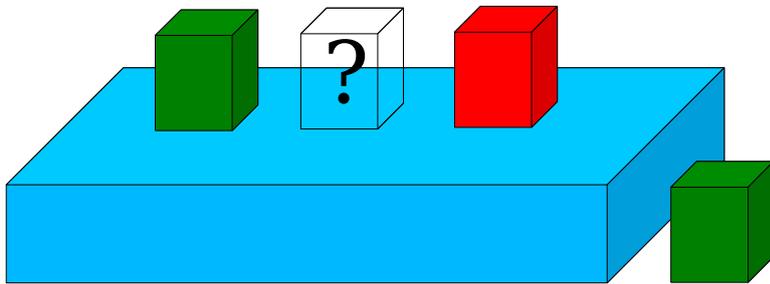
block, green block, green



$$\exists on_table^{-1}. (block \sqcap green \sqcap \exists next_to.(red \sqcap block))$$



Abox Query Answering (4)

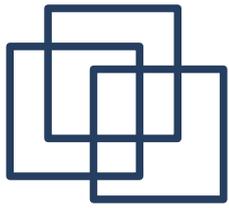


$\text{concept_instances}(\text{table} \sqcap$
 $\exists \text{on_table}^{-1}. (\text{block} \sqcap \text{green} \sqcap$
 $\exists \text{next_to}. (\text{red} \sqcap \text{block}))) = \{t\}$

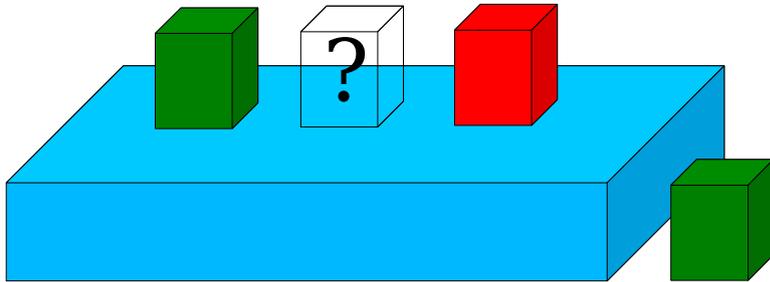
However:

$\text{concept_instances}(\text{block} \sqcap \text{green} \sqcap$
 $\exists \text{next_to}. (\text{red} \sqcap \text{block})) = \{\}$

- Unlike SQL, instance retr. queries can cope with
 - incomplete information (case analysis)
 - have to consider ALL models, not only one (rel.DB)
 - only the existence of such a block is entailed

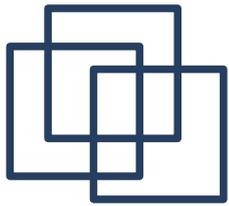


Full Conjunctive Queries

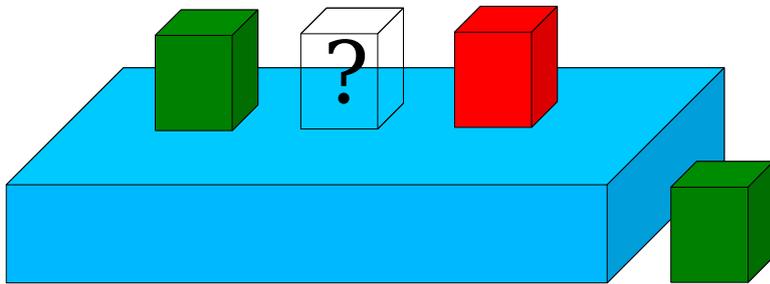

$$\begin{aligned} ans(x) \leftarrow & \text{table}(x), \text{on_table}(y, x), \\ & \text{block}(y), \text{green}(y), \\ & \text{next_to}(y, z), \text{red}(z), \text{block}(z). \end{aligned}$$

Answer: $x = t$

- However, no answer for head $ans(y) \leftarrow \dots$
 - distinguished variables in head: binding must hold in ALL models („certain answer“)
 - other variables: treated as existentially quantified

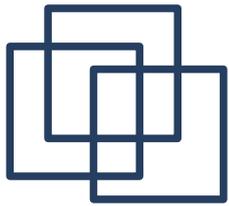


Grounded Conjunctive Queries

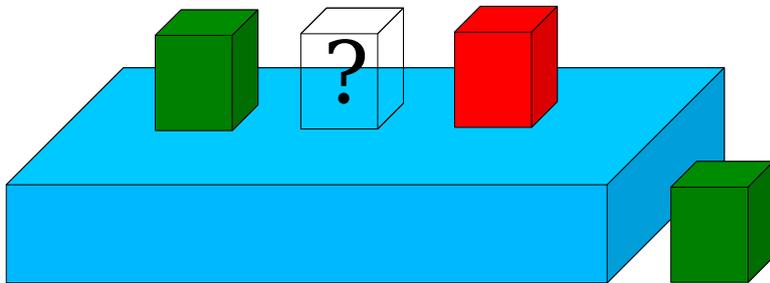

$$\begin{aligned} ans(x) \leftarrow & \text{table}(x), \text{on_table}(y, x), \\ & \text{block}(y), \text{green}(y), \\ & \text{next_to}(y, z), \text{red}(z), \text{block}(z). \end{aligned}$$

Gives no answer in nRQL!

- In grounded conjunctive queries
 - ALL variables are distinguished; a binding is only established iff it holds in ALL models
 - grounding: subst. variables \leftrightarrow entailed assertions



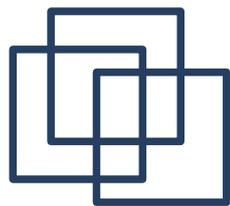
Grounded CQs vs. Full CQs



$$ans(x) \leftarrow (table \sqcap \exists on_table^{-1} \dots)(x)$$

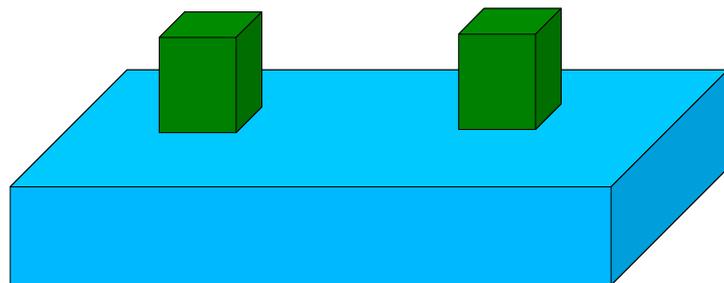
This grounded CQ can be used instead of the full CQ (a simple instance retrieval query)

- This „rolling up“ into nested $\exists R. \exists S. \dots$ works only for non-cyclic queries
 - note that variables may introduce coreferences
 - no automatic rolling up in nRQL



Further Differences with Databases

- Open World Semantics

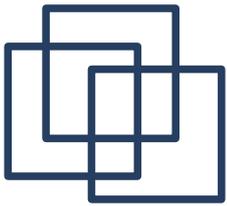


$$ans(x) \leftarrow (\forall on_table.(block \sqcap green))(x)$$

$$ans(x) \leftarrow (\leq_2 on_table)(x)$$

give no answers

- the model / world is not closed - models with additional red blocks or even non-blocks exist, but can be excluded: $t : \leq_2 on_table$
- the two blocks are the only ones \rightarrow all are green
- DB would conclude all blocks are green (CWA/NAF)

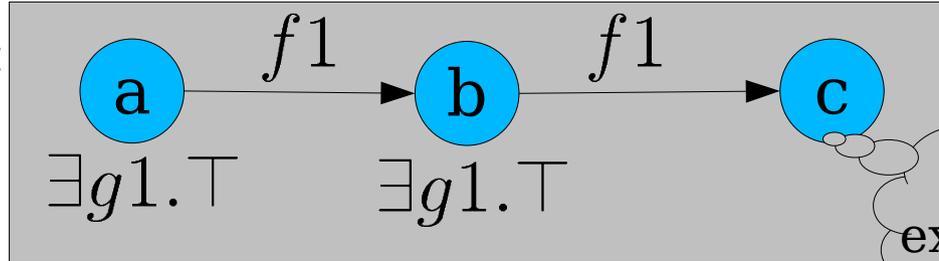


Even GCQs are not so easy...

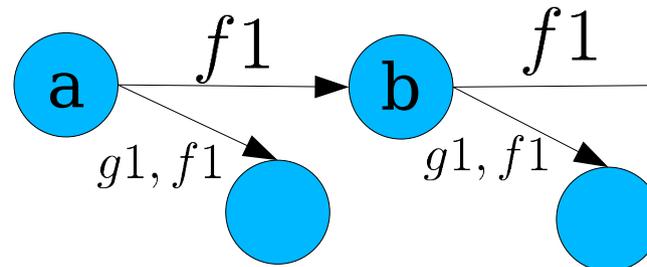
TBox:

functional($f1$)
 transitive($r1$)
 functional($g1$)
 $g1 \dot{\sqsubseteq} f1$
 $g1 \dot{\sqsubseteq} r1$

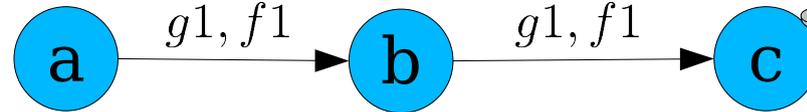
ABox:



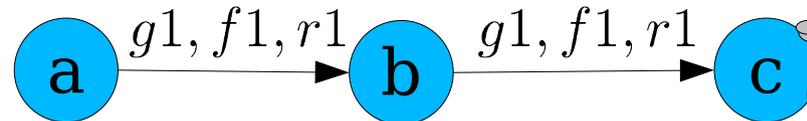
expand existentials



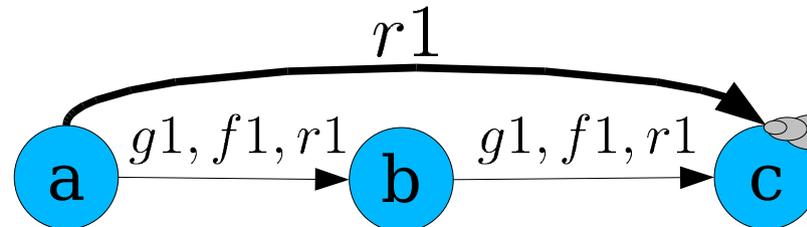
$g1$ has $f1$ as parent role



merge $f1$ successors



$f1$ has $r1$ as superrole

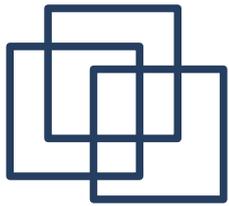


$r1$ is transitive!

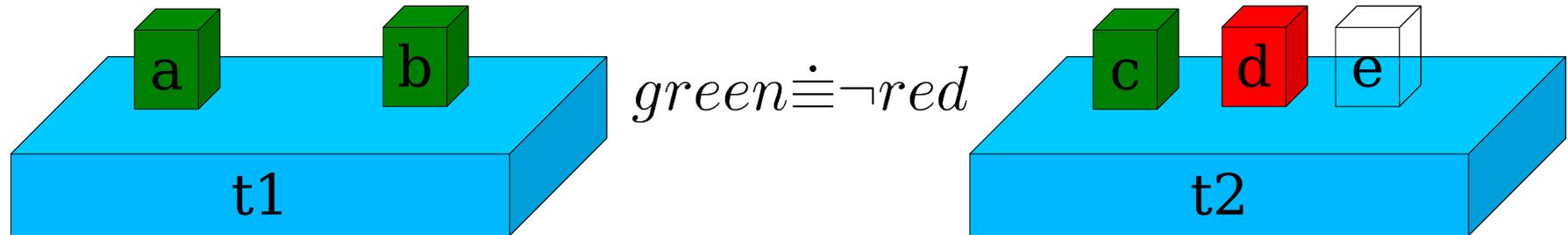
Yes!

?
 $ans(x, y) \leftarrow r1(x, y)$
 ?

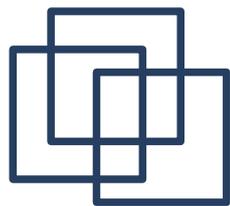




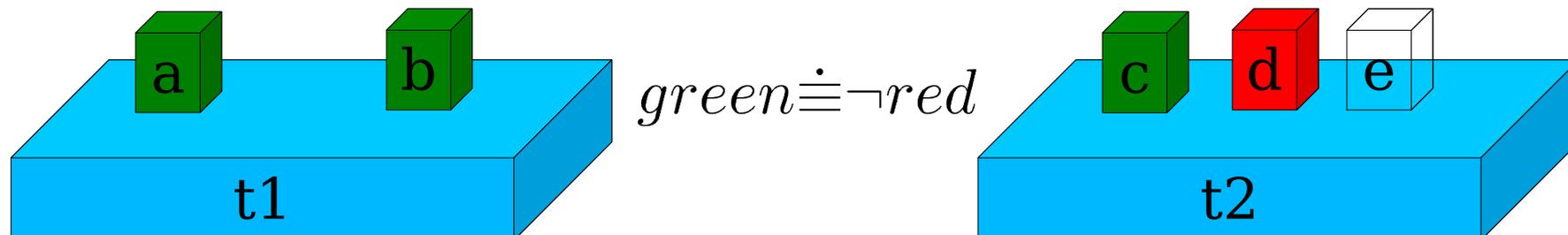
NAF Negation and Projection



- Blocks for which we can prove they are not green:
 $ans(x) \leftarrow block(x), (\neg green)(x) \Rightarrow \{d\}$
- Blocks for which we cannot prove they are green:
 $ans(x) \leftarrow block(x), \backslash green(x) \Rightarrow \{d, e\}$
- Tables f.w.w.c. prove they have only green blocks:
 $ans(x) \leftarrow table(x), (\forall on_table^{-1}.(block \sqcap green))(x) \Rightarrow \{\}$
- f.w.w. CANNOT prove they have NON-green blocks:
 $ans_1(x) \leftarrow table(x), on_table(y, x), \backslash green(x)$
 $ans(x) \leftarrow table(x), \backslash ans_1(x) \Rightarrow \{t1\}$



NAF Negation and Projection (2)



Tables for which we CANNOT prove the have NON-green blocks (all known blocks on table are green):

```
(retrieve (?x)
  (and (?x table)
    (neg (project-to (?x)
      (and (?y ?x on-table)
        (neg (?y green))))))))
(((?X T1)))
```

nRQL is the only and first practical DL QL which can process that kind of queries; new project-to

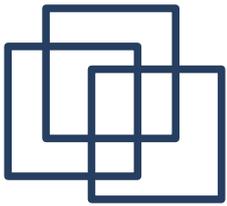
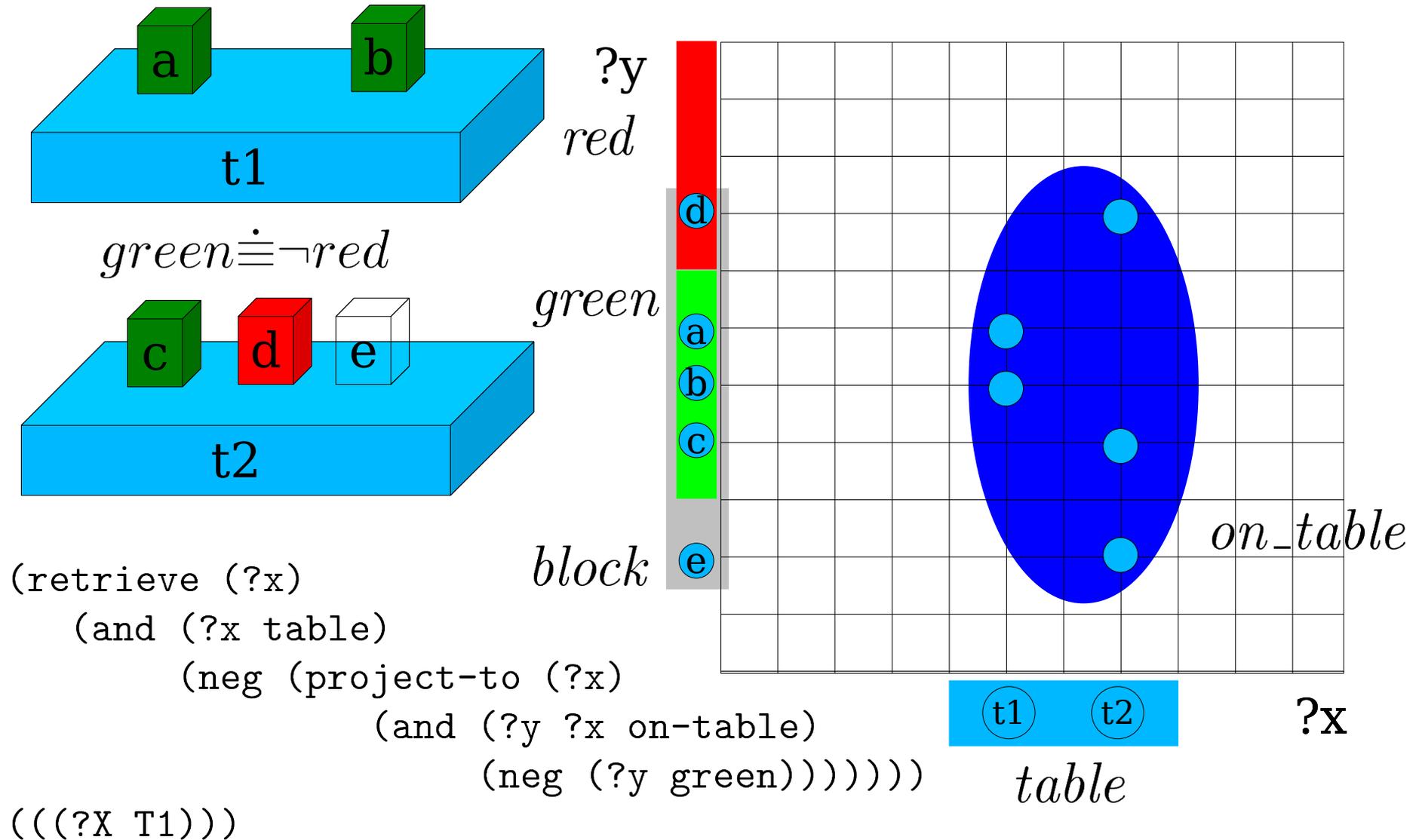


Illustration of nRQL Semantics



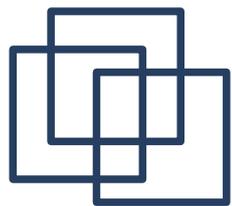
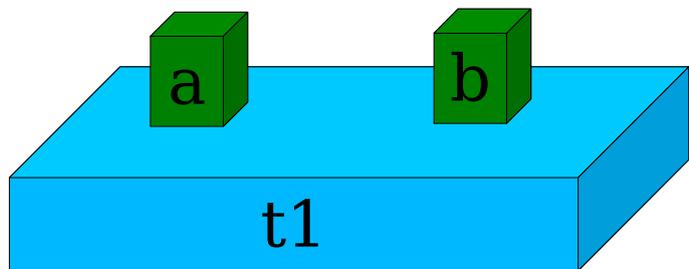
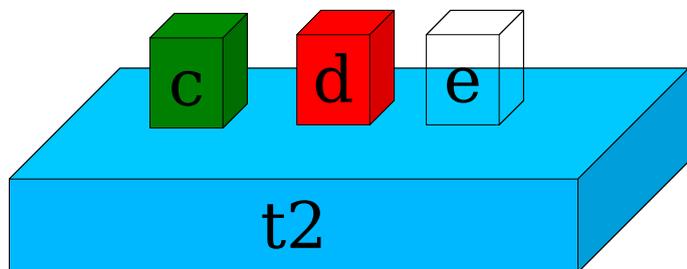


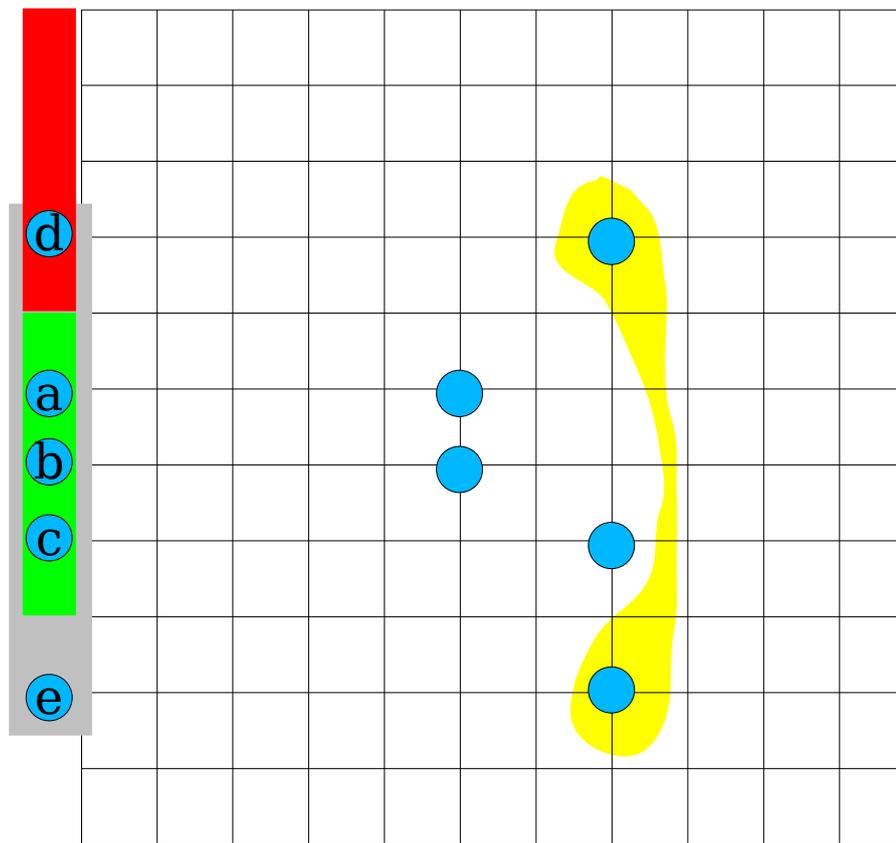
Illustration of nRQL Semantics (2)



$green \doteq \neg red$



?y



```
(retrieve (?x)
  (and (?x table)
    (neg (project-to (?x)
      (and (?y ?x on-table)
        (neg (?y green))))))))
(((?X T1)))
```

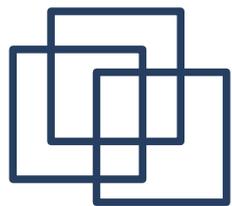
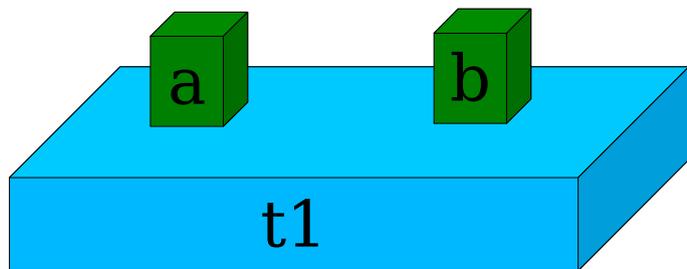
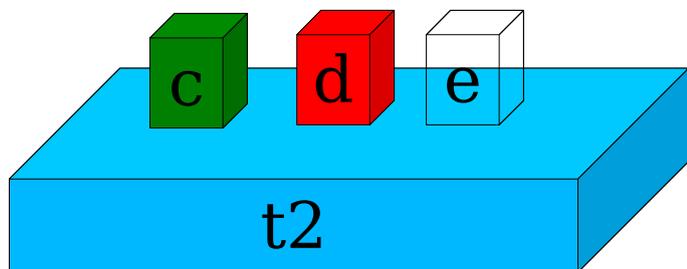


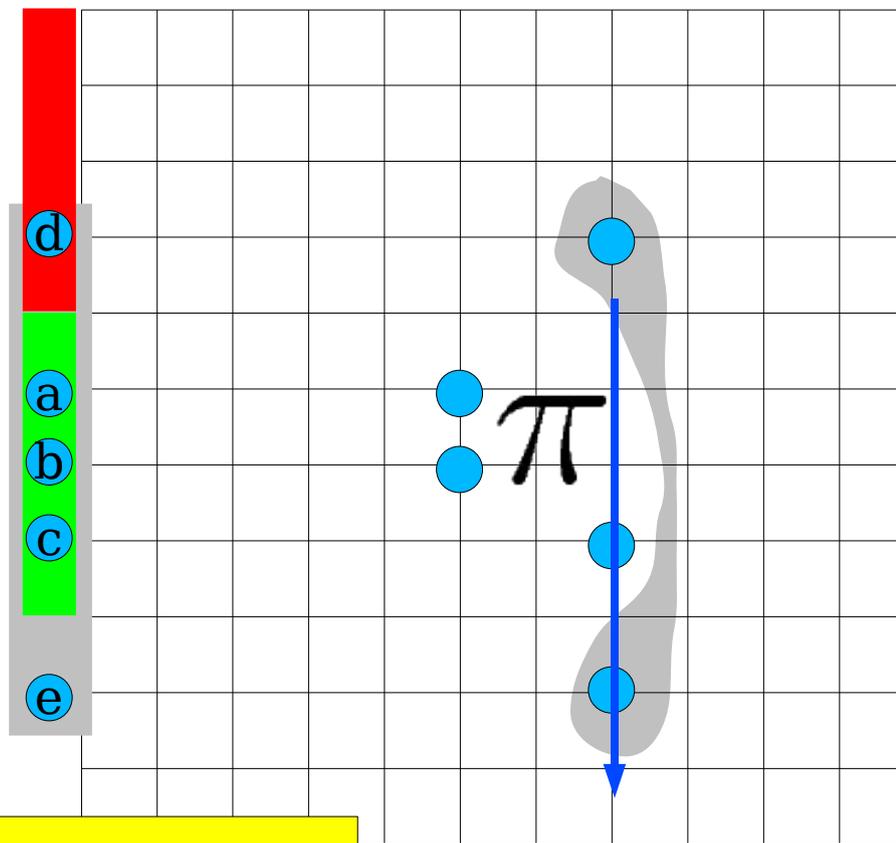
Illustration of nRQL Semantics (3)



$green \doteq \neg red$



?y



```
(retrieve (?x)
  (and (?x table)
    (neg (project-to (?x)
      (and (?y ?x on-table)
        (neg (?y green))))))))
(((?X T1)))
```



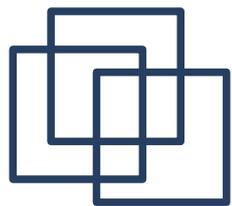
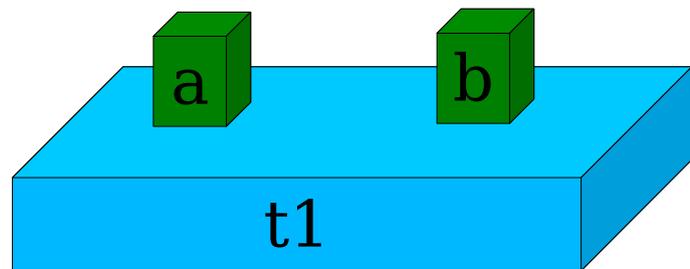
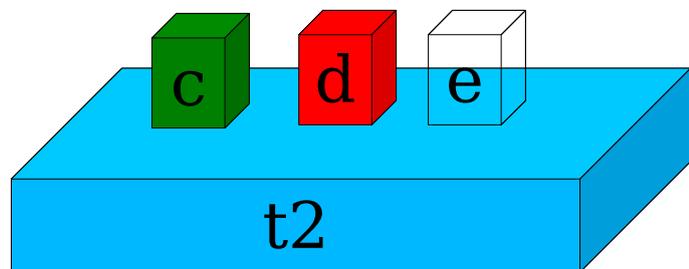


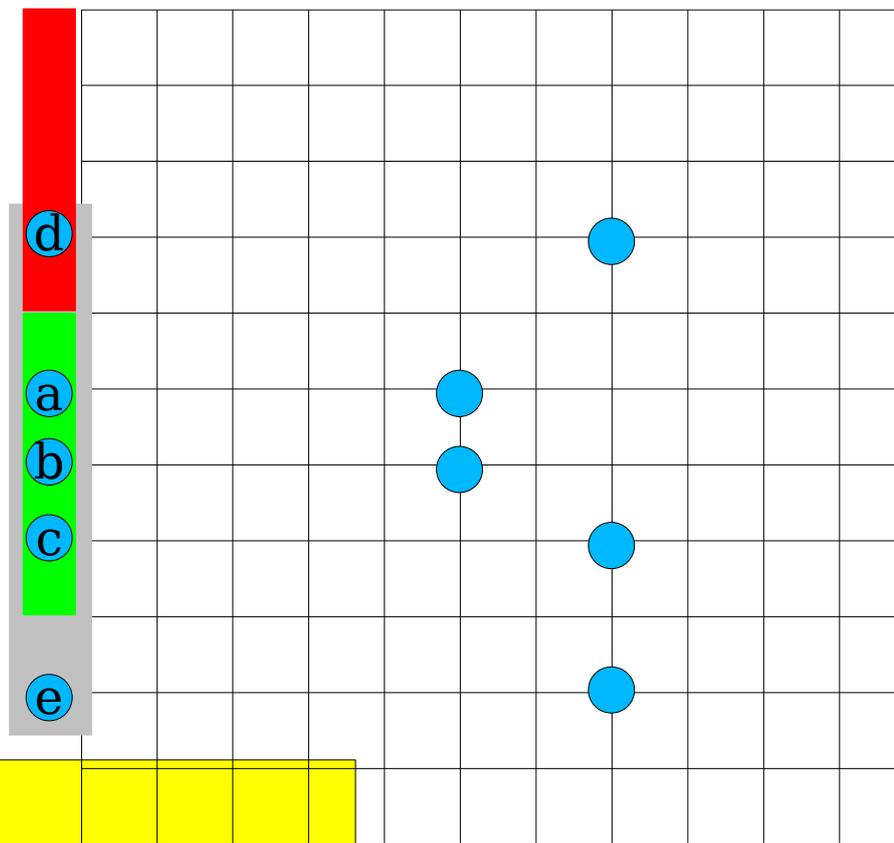
Illustration of nRQL Semantics (5)



$green \doteq \neg red$



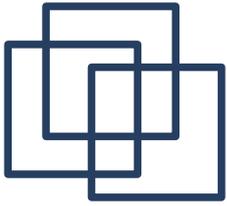
?y



(retrieve (?x)

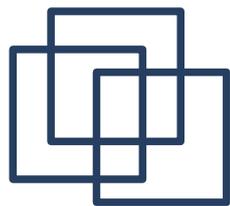
```
(and (?x table)
      (neg (project-to (?x)
                       (and (?y ?x on-table)
                            (neg (?y green))))))))
```

(((?X T1)))



Ontologies & Semantic Web

- Ontologies
 - formal description of a domain of discourse (DOD)
 - „An ontology is an explicit and formal specification of a (shared) conceptualization“
 - formal: prerequisite for computerized reasoning
 - conceptualization: classes and relationship, **abstraction** of DOD (e.g., parent, woman, ...)
 - shared: common understanding of terms
 - common base terms
 - shared conceptual notions (e.g., what constitutes a parent)
 - make these notions **explicit** in a formal description language



Ontologies & Semantic Web (2)

Semantic Web

- today: unstructured HTML documents
- tomorrow: explicit content descriptions („meta data“) for web resources
 - „ontologies for the web“
- Smarter search for pages, services, ...
 - e.g., „recognize“ companies with DL professors!
- Can deal with semantic heterogeneity
 - e.g., semWebCompany ↔ DLCompany

```
<h1>Racer Systems</h1>
```

```
<p>Racer Systems was founded in 2004  
by Volker Haarslev, Ralf M&ouml;ller,  
Kay Hidde, and Michael Wessel.</p>
```

```
⊨ company(Racer Systems)
```

```
⊨ person(V.Haarslev)
```

```
⊨ DL_professor(V. Haarslev))
```

```
<semwebCompany>
```

```
<name>Racer Systems</name>
```

```
<founded>2004</founded>
```

```
<founder>
```

```
<professor>
```

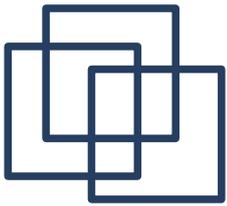
```
<name>Volker Haarslev</name>
```

```
<specialField>DL</specialField>
```

```
</professor>
```

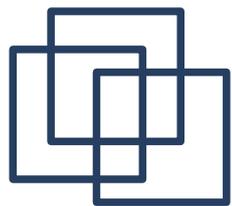
```
...
```

```
</semwebCompany>
```

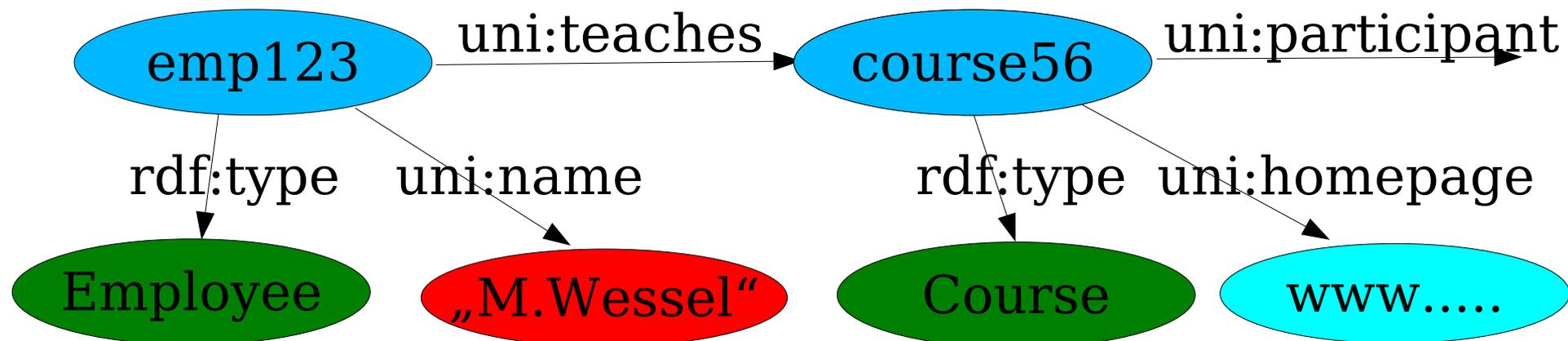


RDF, RDF Schema, SPARQL

- Graph data model
 - Edges = (subject,predicate,object) triples
 - Nodes (subject, object): URIs, literals (e.g., strings)
 - triples „annotate“ Web resources → „meta data“ for the web
 - RDF vocabulary = RDF predicates in a namespace
- Why not XML?
 - XML: trees only
 - RDF (meta) data representation is more canonical (XML too semi-structured, no attribute vs. child element problem → retrieval prob. in XQuery)
 - shallow reasoning in RDFS(++)



RDF, RDF Schema, SPARQL (2)



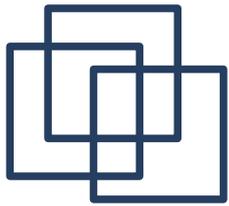
- RDF XML (other syntaxes exist)

```
<rdf:Description rdf:ID="#emp123">
  <rdf:type rdf:resource="#uni:Employee"/>
  <uni:name>M. Wessel</uni:name>
  <uni:teaches rdf:resource="#course56"/>
</rdf:Description>
```

```
<uni:Course rdf:ID="course56">
  <uni:homepage rdf:resource="www...." />
  <uni:participant> ... </uni:participant>
</uni:Course>
```

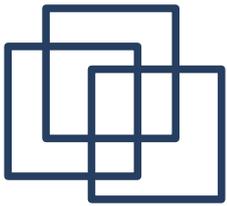
```
select ?x ?y where
{ ?x rdf:type
  uni:Employee .

  ?x uni:teaches
  ?y }
```



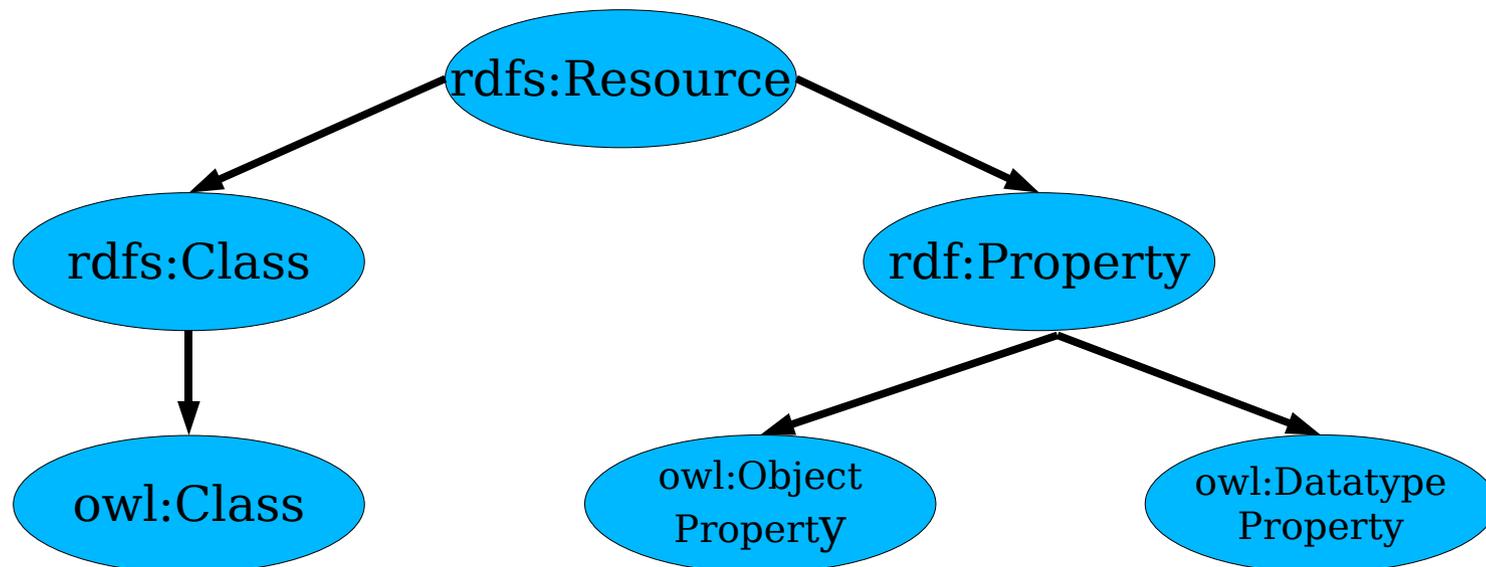
RDF, RDF Schema, SPARQL (3)

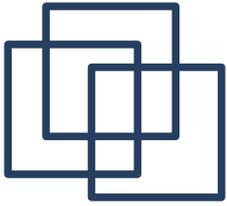
- Expressive means for simple „ontologies“
 - Classes
 - `rdfs:Class` (`rdf:type` already in RDF!)
 - `rdfs:subClassOf`
 - classes and individuals in one graph, no definitions
 - Properties
 - `rdfs:subPropertyOf`, `rdfs:domain`, `rdfs:range`
 - Reification of (s,p,o) triples as nodes:
 - `rdf:subject`, `rdf:predicate`, `rdf:object`
 - Utility properties
 - `rdfs:seeAlso`, `rdfs:comment`, `rdfs:label`, ...
 - RDFS(++): transitive & inverse properties, ...



RDF and OWL

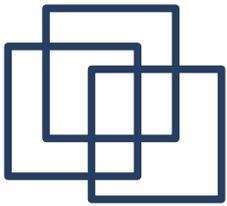
- OWL XML is based on RDF and RDF Schema
 - „ABox“ as in RDF with `rdf:Description`, `rdf:type`
 - but also classes and their descriptions are nodes!
 - OWL specializes RDF Schema predicates, but also restricts possible combinations of predicates to ensure decidability





Species of OWL

- OWL Full
- OWL DL
- OWL Lite
- OWL2
 - Whats new?

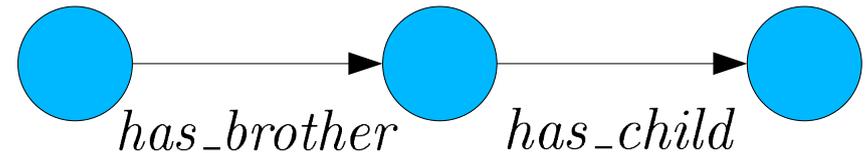


Rules in SWRL and nRQL

- Certain things cannot be expressed in OWL

- no defined roles

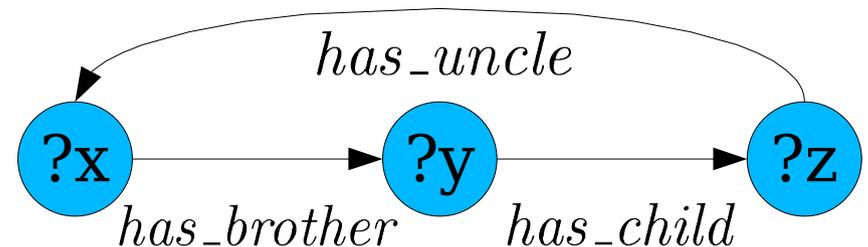
- famous example:



$\forall x, y, z : has_brother(x, y), has_child(y, z) \rightarrow has_uncle(z, x)$

- possible with SWRL rule

- or nRQL rule



(prepare-abox-rule

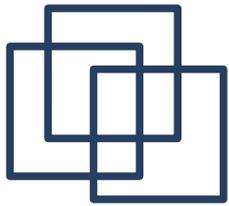
 (and (?x ?y has-brother)

 (?y ?z has-child))

 ((related ?z ?x has-uncle)))

- decidable, if DL-safe (rules are only applied to named individuals in Δ^I)

- more expressive rules in nRQL



Non-monotonic Rules in nRQL

- find *mother* without known children

```
(retrieve (?x)
  (and (?x mother)
    (neg (project-to (?x
      (?x ?y has-child))))))
```

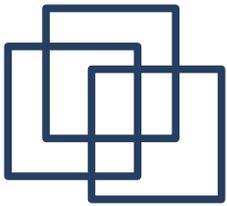
(?x (all has-child bottom))
doesn't work!

- add an explicit child

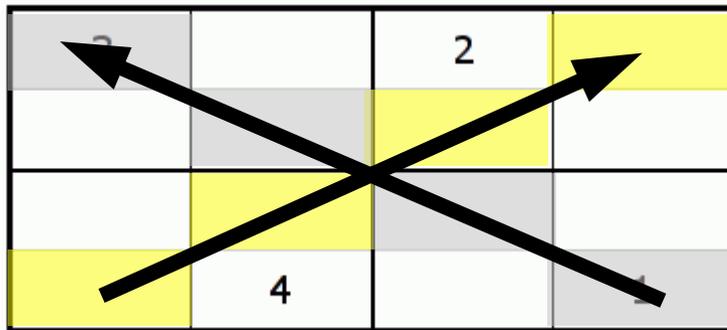
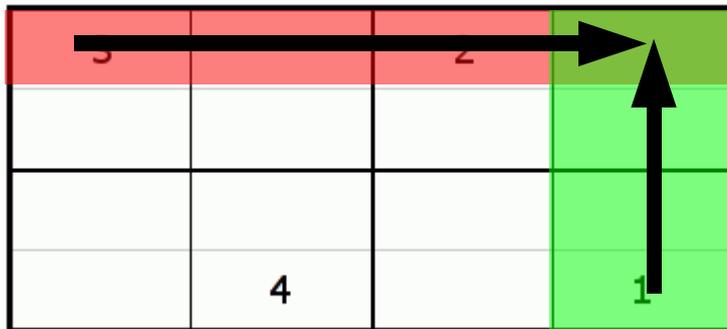
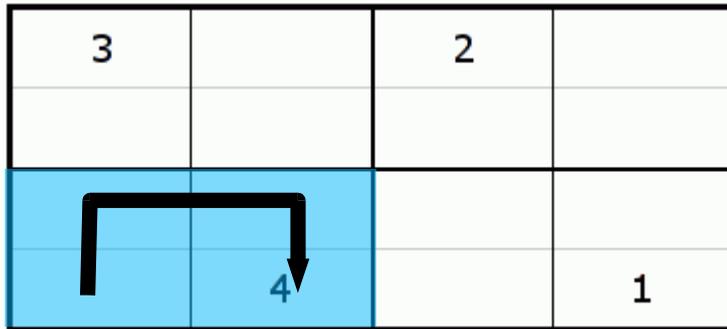
```
(firerule
  (and (?x mother)
    (neg (project-to (?x
      (?x ?y has-child))))))
  ((related ?x (new-ind child-of ?x) has-child)))
```

rule is non-monotonic –
can be applied at most once

- not possible with SWRL
- no automatic rule application strategy in nRQL



Application: Sudoku



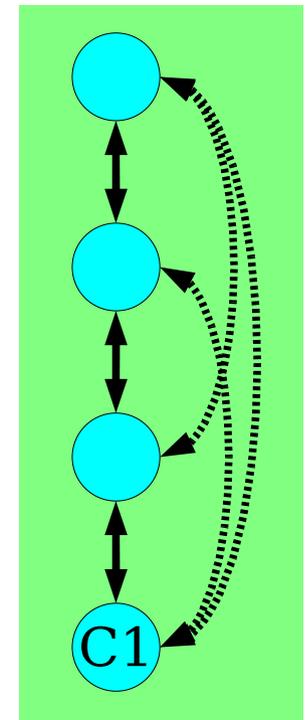
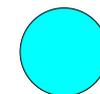
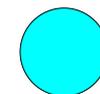
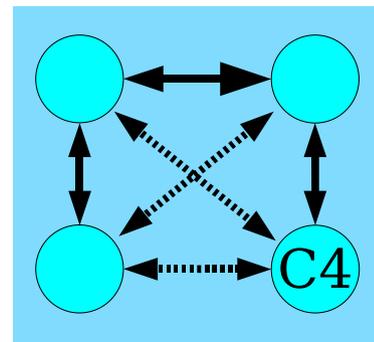
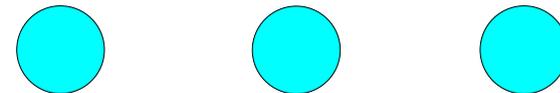
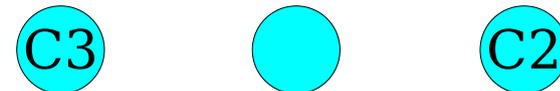
pairwise_disjoint(C_1, C_2, C_3, C_4)

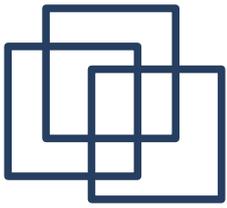
$\top \sqsubseteq (C_1 \sqcup C_2 \sqcup C_3 \sqcup C_4) \sqcap$

$(C_1 \rightarrow \forall R. \neg C_1) \sqcap (C_2 \rightarrow \forall R. \neg C_2) \sqcap$

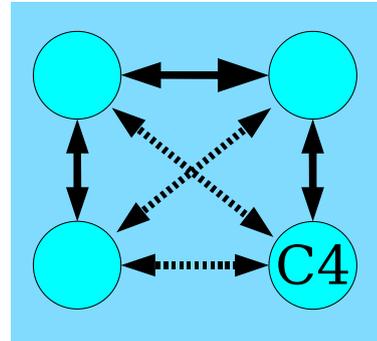
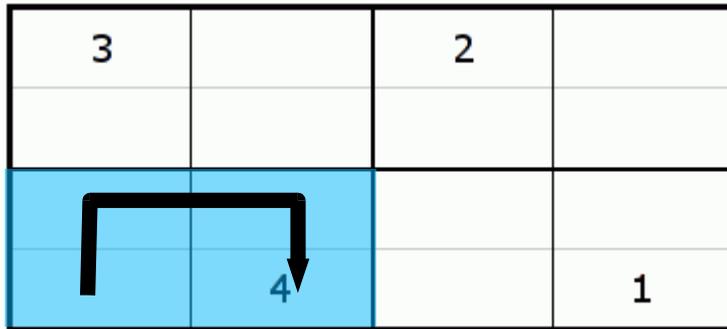
$(C_3 \rightarrow \forall R. \neg C_3) \sqcap (C_4 \rightarrow \forall R. \neg C_4) \sqcap$

...



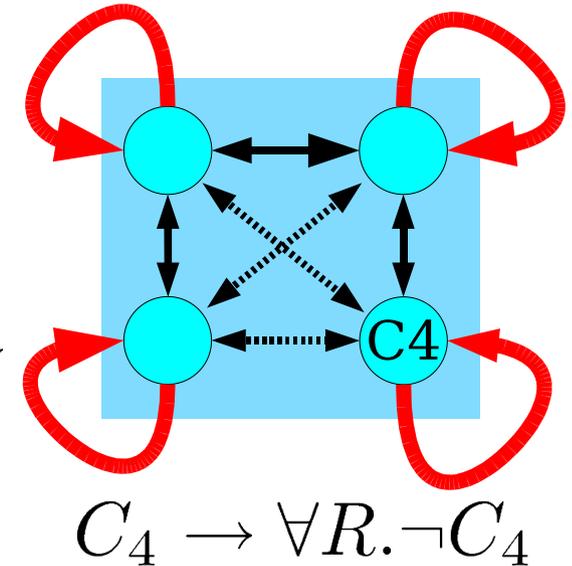


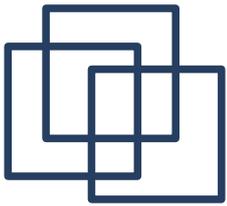
Sudoku (2)



ABox construction

- by hand (OK for 2x2, but for 3x3 ?)
- transitive + symmetric role? →
- use different „backward“ role for other direction, qualification over common parent role
- use a rule to create inverse edges





Sudoku (3)

3		2	
			1

A diagram of a 4x4 grid with a blue shaded 2x2 region in the bottom-left corner. A black arrow starts at the top-left of this region, moves right, then down, then right again, ending at the cell containing the number 4.

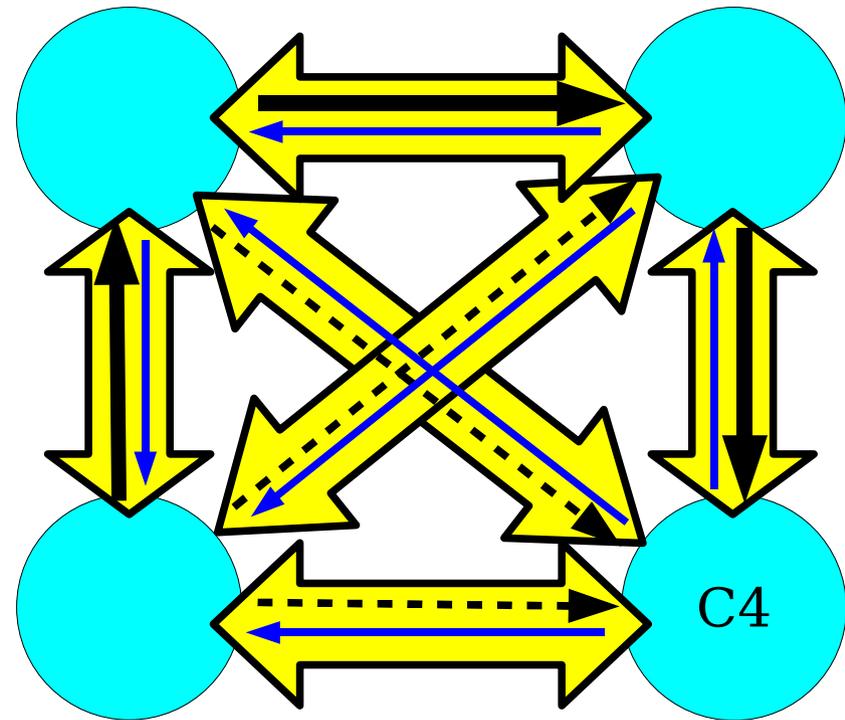
$$Q_1 \dot{\subseteq} R$$

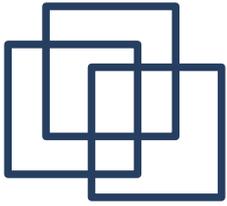
$$Q_2 \dot{\subseteq} R$$

transitive(Q_1)

transitive(Q_2)

$$Q_1(x, y) \rightarrow Q_2(y, x)$$





Sudoku (4)

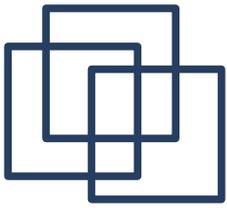
3	1	2	4
4	2	1	3
1	3	4	2
2	4	3	1

```
(retrieve (?x) (?x c1))  
(((?X I41)) ((?X I12)) ((?X I24)) ((?X I33)))
```

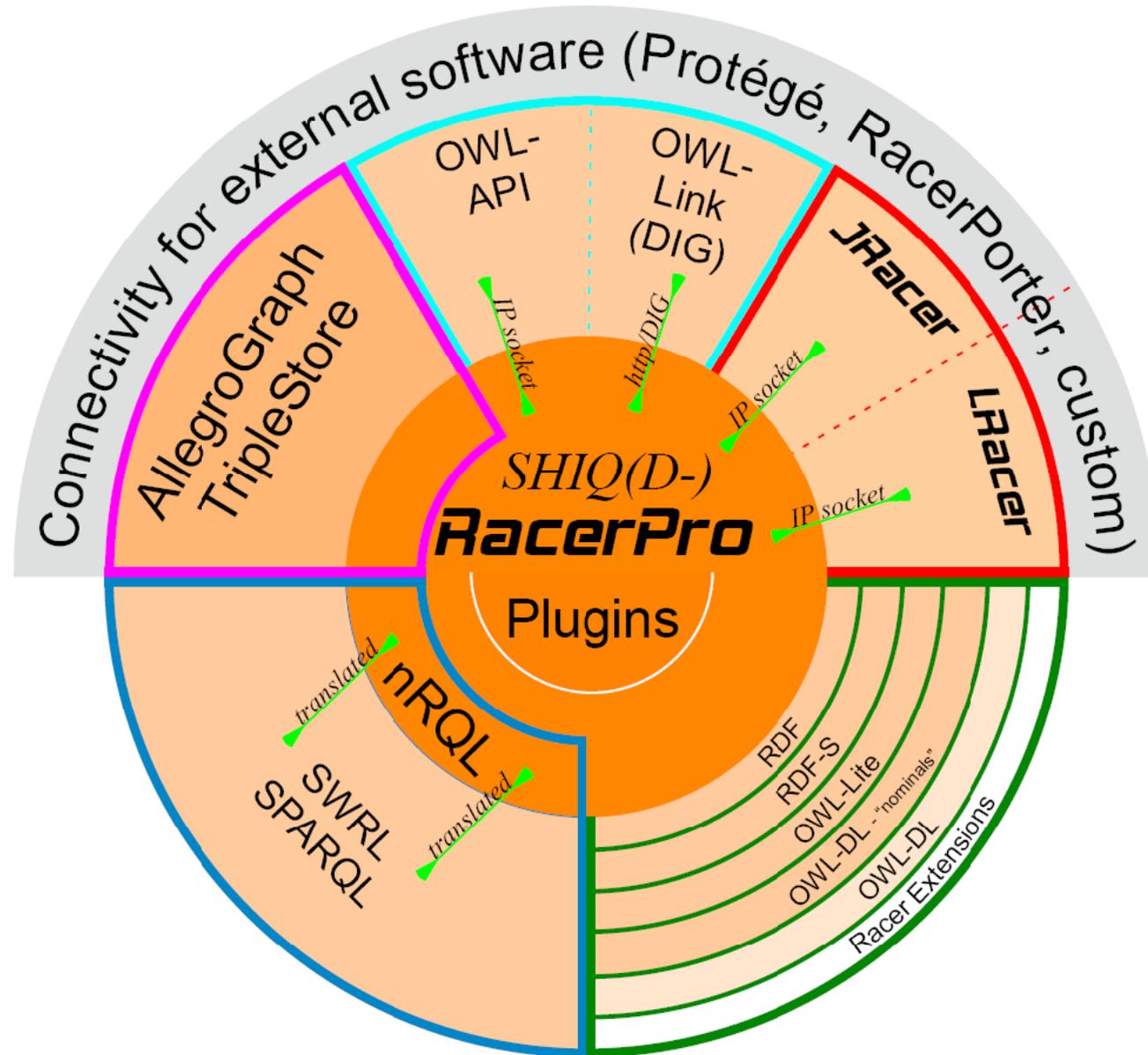
```
(retrieve (?x) (?x c2))  
(((?X I34)) ((?X I42)) ((?X I11)) ((?X I23)))
```

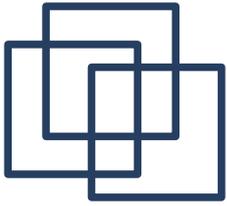
```
(retrieve (?x) (?x c3))  
(((?X I14)) ((?X I31)) ((?X I43)) ((?X I22)))
```

```
(retrieve (?x) (?x c4))  
(((?X I21)) ((?X I13)) ((?X I44)) ((?X I32)))
```



RacerPro - Architecture





RacerPro & Friends

- To obtain a free educational trial version of RacerPro, please visit

`http://www.racer-systems.com/products/download/education.phtml`

- Demo Session
 - Protege 3.4 Beta (OWL)
 - RacerPorter / people+pets.owl